FEB 26 1926

VOLUME LXIII NUMBER 1

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

JANUARY 1926

ON THE WINGED LINES IN THE SOLAR SPECTRUM
Charlotte E. Moore and Henry Norris Russell 1

NOTES ON LINES IN THE SPECTRA OF RED STARS - - - - - *Paul W. Merrill* 13

THE MAGNESIUM HYDRIDE BAND SPECTRUM - *William W. Watson and Philip Rudnick* 20

SOME STUDIES OF THE STARK EFFECT - - - - *Charles E. Deppermann* 33

THE ATMOSPHERE OF MARS - - - - - - - - *Donald H. Menzel* 48

PRELIMINARY ORBIT OF THE LONG-PERIOD SPECTROSCOPIC BINARY 37 ϕ^1 ORIONIS - - - - - - - - - - - - - *Otto Struve* 60

INTERNAL MOTIONS OF MESSIER 33 - - - - - - - - *Knut Lundmark* 67

ERRATA - 72

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

THE CAMBRIDGE UNIVERSITY PRESS, London
THE MARUZEN-KABUSHIKI-KAISHA, Tokyo, Osaka, Kyoto, Fukuoka, Sendai
THE COMMERCIAL PRESS, LIMITED, Shanghai

January 1926 Vol. LXIII, No. 1

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

WITH THE COLLABORATION OF

WALTER S. ADAMS, Mount Wilson Observatory
JOSEPH S. AMES, Johns Hopkins University
ARISTARCH BELOPOLSKY, Observatoire de Poulkova
WILLIAM W. CAMPBELL, Lick Observatory
HENRY CREW, Northwestern University
CHARLES FABRY, Université de Paris
ALFRED FOWLER, Imperial College, London
CHARLES S. HASTINGS, Yale University
HEINRICH KAYSER, Universität Bonn
ALBERT A. MICHELSON, University of Chicago
HUGH F. NEWALL, Cambridge University
CARL RUNGE, Universität Göttingen
HENRY N. RUSSELL, Princeton University
SIR ARTHUR SCHUSTER, Twyford
FRANK SCHLESINGER, Yale Observatory

The Astrophysical Journal is published by the University of Chicago at the University of Chicago Press, 5750 Ellis Avenue, Chicago, Illinois, during each month except February and August. ¶ The subscription price is $6.00 a year; the price of single copies is 75 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶ Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Dominican Republic, El Salvador, Argentina, Bolivia, Brazil, Colombia, Costa Rica, Ecuador, Guatemala, Honduras, Nicaragua, Peru, Uruguay, Paraguay, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, and Spain. ¶ Postage is charged extra as follows: for Canada, 30 cents on annual subscriptions (total $6.30); on single copies, 3 cents (total 78 cents); for all other countries in the Postal Union, 50 cents on annual subscriptions (total $6.50), on single copies 5 cents (total 80 cents). ¶ Patrons are requested to make all remittances payable to The University of Chicago Press in postal or express money orders or bank drafts.

The following are authorized to quote the prices indicated:

For the British Empire: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, £1 12*s.* 6*d.* each; single copies, including postage, 4*s.* each.

For China: The Commercial Press, Ltd., Paoshon Road, Shanghai. Yearly subscriptions, $6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, on yearly subscriptions 50 cents, on single copies 5 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business Correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Yerkes Observatory, Williams Bay, Wisconsin.

The cable address is "University, Chicago."

The articles in this Journal are indexed in the *International Index to Periodicals*, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Post-office at Chicago, Ill., under the Act of March 3, 1879.
Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 15, 1918.

PRINTED IN THE U.S.A.

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME LXIII JANUARY 1926 NUMBER 1

ON THE WINGED LINES IN THE SOLAR SPECTRUM

By CHARLOTTE E. MOORE and HENRY NORRIS RUSSELL

ABSTRACT

The most conspicuously *winged lines* in the solar spectrum were selected by an independent examination of the Mount Wilson and Rowland photographs, and classified according to the degree of widening. The 138 widened lines are *due to eleven elements, Fe, Mg, Ca, Ni, H, Ti, Na, Al, Si, Cr,* and *Sr.* Iron gives most of the lines (89); *Ca+*, the strongest (H and K).

Energy-level of origin.—The widened lines all originate in low energy-levels, those arising from the lowest levels being the most widened. Strong components of multiplets are more widened than weak.

Relative abundance of elements.—Using a rough criterion for the "total widening" of the lines, the order of the elements is as given above. This is closely parallel to that of the relative abundance in the composition of the earth, the principal exceptions being explainable.

The main *cause of widening* is shown by this evidence to be almost entirely the *abundance* in the solar atmosphere of atoms which are in a condition to absorb the line in question. Widening in the laboratory shows little correlation with widening in the sun, and is probably due to quite different causes.

Tables of the widened lines and their spectroscopic relations are given.

The solar spectrum contains a large number of widened or hazy lines, of which the strong winged lines stand out conspicuously. An examination has been made of the Rowland and Mount Wilson photographs of the solar spectrum, and the hazy lines classified. The strong winged lines and those fainter lines which are conspicuously hazy were called "A"; the moderately hazy lines, "B"; and those lines which do not appear to be strictly sharp, "C." This paper discusses only the 138 lines of class A.

These lines have been subdivided into four groups according to the degree of widening: group 1 represents the least widened lines, while group 4 includes only H and K.

Rowland's map was used from λ 3000 to λ 6950, Meggers' map (issued by the Allegheny Observatory) from this point to λ 9000, and the enlarged Mount Wilson sun-spot map from λ 3900 to λ 6600.

The accuracy of classification is considerably greater to the red of λ 3900 than to the violet, where only Rowland's map was available; but care has been taken to keep the classification as nearly homogeneous as possible, and to guard against spurious widening due to faint adjacent lines. A complete list of the widened lines was drawn up before any attempt at identification was made. Later revisions added one line that had been overlooked, and changed the classification by a unit in three or four cases.

Of the 138 winged lines, all are of known chemical origin, eleven different elements being represented. For all of these elements the spectra have been analyzed by various investigators, so that the energy relations of the lines are known.

In Table I, the elements to which the widened lines are due are listed in the order of their atomic numbers, and the lines tabulated in the order of the energy-levels from which they arise. In column 1 the excitation potential (E.P.) in volts is given, followed by the series notation of the multiplet to which the lines belong, intersystem combinations being in parentheses. These data are taken from Russell's list[1] with a few additions.[2] The figures below the multiplet designation in column 1 indicate the total number of lines in the multiplet and the intensities of the two strongest lines which have not been classified as widened. For example, in the $1f^5 - 2d'^5$ multiplet of *Fe* there are twelve lines, and the intensities of the two strongest lines omitted in selecting the widened lines are 10 and 8. Column 2 gives the wave-length on the international scale, and in columns 3 and 4 A. S. King's intensity and temperature class are given when known. When King's data are not available the intensity is placed in parentheses. In column 5 the Rowland intensity is given, and in column 6 the estimated degree of widening is shown.

[1] *Astrophysical Journal*, **61**, 223, 1925.

[2] *Fe:* Walters, *Journal of the Optical Society of America*, **8**, 245, 1924, supplemented by unpublished work by the writers. *Si:* McLennan and Shaver, *Transactions of the Royal Society of Canada*, III, **18**, 1, 1924. *Ni:* Walters, *Journal of the Washington Academy of Sciences*, **15**, 88, 1925; Bechert and Sommer, *Annalen der Physik*, **77**, 351, 1925.

TABLE I

WINGED LINES IN THE SOLAR SPECTRUM

E.P. MULTIPLET	λ I.A.	INT. (King)	CLASS (King)	INT. (Rowland)	WIDENING
		Hydrogen, *H*			
10.15					
2P−3D *Hα*	6562.79	(10)	.....	40	3
2P−4D *Hβ*	4861.33	(9)	.....	30	3
2P−5D *Hγ*	4340.46	(8)	.....	20N	2
2P−6D *Hδ*	4101.74	(7)	.....	40N	2
		Sodium, *Na*			
0.0					
(D_1)	5895.93	(10R)	.....	20	2
$1s^2-1p^2(D_2)$	5889.96	(8R)	.....	30	3
		Magnesium, *Mg*			
2.67					
	5183.67	125	II	30	3
$1p^3-1s^3$	5172.70	80	II	20	3
	5167.38	40	II	15	1
	3838.29	100r	II	25	3
$1p^3-2d^3$	3832.31	80r	II	15	3
	3829.36	40	II	10	2
$1p-2s^3$	3336.69	20	III	8N	1
3; 10	3332.14	15	III	3	1
4.33					
1P−2D	8806.75	(8)	.....	.....	2
1P−3D	5528.42	10	V	8	1
1P−4D	4703.00	40	V	10	1
1P−6D	4167.39	10n	III?	8	1
		Aluminium, *Al*			
0.0					
	3961.54	(10R)	.....	20	2
$1p^2-1s^2$	3944.03	(10R)	.....	15	1
		Silicon, *Si*			
1.90					
?	3905.51	.....	.....	12	2
		Calcium, *Ca*			
0.0					
1S−1P	4226.73	500R	I	20d?	3

TABLE I—*Continued*

E.P. Multiplet	λ I.A.	Int. (King)	Class (King)	Int. (Rowland)	Widening
	Ionized Calcium, Ca^+				
0.0					
$1s^2-1p^2$ (H)	3968.47	350R	II	700	4
(K)	3933.66	400R	II	1000	4
1.71					
	8662.11	(9)	V	.....	3
$1d^2-1p^2$	8542.15	(10)	V	.....	3
	8498.00	(8)	V	.....	3
	Ionized Titanium, Ti^+				
0.0					
	3383.76	40r	III	3	1
a^4F-a^4G	3372.80	30r	II	5	1
9; 40, 15	3349.41	40r	II	7	2
a^4F-a^4D	3088.03	60r	III	7d?	1
9; 45, 40	3075.22	40r	III	3	1
	Chromium, Cr				
0.0					
$1s^7-2p^7$	3578.70	200R	II	10	2
3; 160, 140	.....	.....	.....	.....	.....
	Iron, Fe				
0.0					
	3927.92	30R	I	8	2
	3922.92	25R	I	12d?	1
	3920.26	20r	I	10	1
	3899.71	30R	I	8	1
$1d^5-1d'^5$	3895.66	25r	I	7	1
12; 25,* 8	3886.29	40R	I	15	2
	3878.58	100r	II	7Nd?	1
	3859.91	300R	I	20	3
	3856.37	50r	IA	8	1
	3824.44	50r	IA	6	1
	3748.26	60R	IA	10	1
	3745.90	40r	IA	6	1
	3745.56	100R	I	8	2
	3737.14	150R	I	30	3
$1d^5-1f'^5$	3733.32	40r	IA	7d?	1
12; 20, 10	3722.57	50r	IA	10	1
	3719.94	250R	I	40	3
	3705.57	100r	I	9	2
	3679.92	40r	IA	9	1

TABLE I—*Continued*

E.P. Multiplet	λ I.A.	Int. (King)	Class (King)	Int. (Rowland)	Widening
		Iron—*Continued*			
	3490.58	100r	I	10N	1
	3476.71	40	I	8	1
$1d^5-1p'^5$	3475.45	70r	I	10	1
9; 60, 40	3443.88	50r	I	8N	1
	3440.99	75R	I	15	2
	3440.61	150R	I	20	3
	3059.09	100R	I	20	1
$1d^5-2d'^5$	3047.61	100R	I	20N	2
0†	3037.39	80R	I	10N	1
	3025.85	50R	I	2N	1
$(1d^5-1p'^5)$	3024.04	15r	IA	4Nd?	1
7; 40, 20					
0.94					
	5371.50	50	IB	3	1
$1f^5-1d'^5$	5328.04	50	IB	8d?	1
12; 40, 40	5269.54	60	IB	8d?	1
	3887.05	15	I	7	1
	3878.02	60	II	8	1
	3872.50	60	II	6	1
	3865.53	30	II	7	1
$1f^5-2d'^5$	3849.97	40	II	10	1
12; 10, 8	3840.44	80r	II	8	2
	3834.23	100r	II	10	3
	3825.89	200R	II	20	3
	3820.43	250R	II	25	3
	3799.55	50	II	7	1
	3798.51	40	II	6	1
	3795.00	60	II	8	1
	3787.88	50	II	9	1
$1f^5-2f'^5$	3767.19	80r	II	8	2
13; 75, 50	3763.79	100r	II	10	2
	3758.23	150R	II	15	3
	3749.49	200R	II	20	3
	3734.87	300r	II	40	3
	3647.84	100R	I	12	2
	3631.46	125R	I	15	2
$1f^5-1g'^5$	3618.77	125R	I	20	2
11; 30, 30	3608.87	100r	I	20	2
	3581.20	250R	I	30	3
$(1f^5-1g'^5)$	3570.10	100R	I	20	3
8; 30, 30	3565.38	60r	II	20	2
	3099.97	15	II	3	1
$1f^5-3d'^5$	3099.90	20	II	3	1
12; 30, 20	3075.73	25r	II	3	1
	3057.45	40R	II	20	1

TABLE I—*Continued*

E.P. Multiplet	λ I.A.	Int.	Class	Int.	Widening
		King		Rowland	
		Iron—*Continued*			
1.54					
$1f^3-1d'^3$	5167.49‡	40	II	5	1
5; 40, 30					
	4415.13	20	II	8	1
$(1f^3-1g'^5)$	4404.75	30	II	10	1
8; 15, 10	4383.55	45r	II	15	2
	4325.77	35	II	8	2
$1f^3-1g'^3$	4307.91	35	II	6	1
6; 25, 10	4271.76	35	II	15	2
	4202.03	30	I	8	1
	4143.87	30	I	15	1
	4132.06	25	II	10	1
$1f^3-2f'^3$	4071.75	40	II	15	2
7; 30§, 25	4063.60	45	II	20	2
	4045.82	60r	II	30	2
	3888.52	20	II	5	1
$1f^3-2d'^3$	3841.05	80r	II	10	2
6; 20, 10	3827.83	75r	II	8	2
	3815.84	100r	II	15	3
2.19					
$1p^5-3d'^5$	4528.62	18	II	8	1
9; 12, 12					
2.46					
	4260.49	35	III	10	1
$1d'^7-md^7$	4235.95	25	III	8	1
12; 25, 20					
$1d'^7-nd^7$	3225.79‖	25	III	3	1
2.84					
$1f'^7-md^7$	4957.61	60	III	8	2
15; 30, 25	4920.52	60	III	10	1
	4891.51	50	III	8	1
2.96					
$1p'^7-md^7$	5232.95	40	III	7	1
9; 30, 30					
3.25					
$1d'^5-md^5$	5324.20	30	IV	7	1
12; 18, 12					
3.38					
$1f'^5-md^5$	5615.66	50	IV	6	1
12; 40, 30					
		Nickel, *Ni*			
0.00					
$1f^3-1d^3$	3369.57	80R	II	6	1
6; 30, 25					
0.12					
$1d'^3-1p'^3$	3524.54	200R	II	20	2
6; 80, 60	3492.97	150R	II	10N	1

TABLE I—*Continued*

E.P. Multiplet	λ I.A.	Int. King	Class King	Int. Rowland	Widening
	Nickel—*Continued*				
$1d'^3-1f'^3$ 6; 40, 15	3461.66	125R	II	8	1
$1d'^3-2f'^3$ 6; 20, 8	3515.07	150R	II	12	1
	3458.46	125R	II	8	1
	3433.58	70	II	8d?	1
	3414.77	150R	II	15	2
$1d'^3-3f'^3$ 6; 60, 50	3101.56	100R	II	4N	1
	3050.82	100R	II	5	1
	3037.93	60R	II	3	1
0.42					
$1D'-1p'$ 2; 25	3831.70	20	II	6	1
$1D'-1F'$	3619.40¶	150R	II	8	1
$1D'-1D$	3566.38	100R	II	10	1
$1D'-1P'$	3380.58	80R	II	6N	1
	Ionized Strontium, Sr^+				
0.0					
$1s^2-1p^2$ 2; 300	4077.71	400r	II	8	1

* λ 3930.30—in wing of K.
† Lines to the violet of λ 3022 disregarded as unobservable in the sun.
‡ Probably not widened. Difficult blend with *Mg* line.
§ λ 3969.26. In wing of H.
|| Multiplet not complete.
¶ ? In wing of *Fe* line.

The data are summarized in Table II, which gives for each element the number of lines showing each degree of widening, the whole number of widened lines, and the sum of the numbers which express the individual widening. The order is that of this last quantity, which may be taken as a crude index of the general contribution of the element to the widened solar lines.

A conspicuous correlation between the level of origin and the degree of widening is evident among the iron lines. Table III gives for these the excitation potential and multiplet designation, as in Table I; and the numbers of lines in each multiplet which are widened to the various degrees (o denoting a line not notably widened). Multiplets which contain no widened lines are not listed. In

the groups which arise from the lowest energy-levels, almost all the lines are widened, the exceptions being nearly always the weakest lines of the multiplet. Most of these are usually somewhat widened and are classified as "B." Passing to higher levels of origin the weaker lines drop out, until only the strongest line of the multiplet shows enough widening to be called "A." The correlation between the intensity of the lines and the degree of widening is also close, as may be seen in Table I.

TABLE II

	Degree of Widening				Total	
	4	3	2	1	No.	Widening
Fe	.....	13	23	53	89	138
Mg	.....	4	2	6	12	22
Ca+	2	3	.....	.....	5	17
Ca	.....	1	.....	.....	1	3
Ni	.....	.....	2	13	15	17
H	.....	2	2	.....	4	10
Ti+	.....	.....	1	4	5	6
Na	.....	1	1	.....	2	5
Al	.....	.....	1	1	2	3
Si	.....	.....	1	.....	1	2
Cr	.....	.....	1	.....	1	2
Sr+	.....	.....	.....	1	1	1

The widened lines of nickel all originate in the group of low levels discovered by Walters. Only the stronger components of the multiplets appear in the list.

The magnesium lines, which are also listed in Table III, show a decided tendency for the earlier members of a series to be more strongly winged than the later. For the remaining elements the lines are few and Table I suffices.

It may be added that certain arc lines of calcium, though not widened in the solar spectrum, are conspicuously winged in the sunspot spectrum. These are λλ 6162.18, 6122.22, 6102.72 ($1p^3 - 1s^3$), for which E.P. = 1.88, and 6439.09 (the strongest line of $1d^3 - 1f''^3$), for which E.P. = 2.53.

Finally, it is worthy of special attention that the type of widening exhibited by the hydrogen lines differs from that of the rest. All the other winged lines have a central core, as dark as, and (except for H and K) not much wider than, strong lines which are not

winged. The hydrogen lines are wide and diffuse, and obviously not so dark at the center as other lines, as is well seen on the Mount Wilson map.

The material here collected appears to permit of a fairly definite determination of the origin of the widening of solar lines. In the first place, it is clear that the causes of solar widening are different

TABLE III

E.P. Multiplet	3	2	1	0
	Magnesium			
(2.67)				
$1p^3-1s^3$	2	.....	1	.....
$-2d^3$	2	1	.....	.....
$-2s^3$	.....	.....	2	1
(4.33)				
1P−2D	.....	1	.....	.....
−3D	.....	.....	1	.....
−4D	.....	.....	1	.....
−6D	.....	.....	1	.....
	Iron			
(0.0)				
$1d^5-1d'^5$	1	2	7	2
$-1f'^5$	2	2	5	3
$-1p'^5$	1	1	4	3
$-2d'^5$	.....	1	3	(0)*
$-(1p'^3)$	.....	.....	1	6
(0.94)				
$1f^5-1d'^5$	.....	.....	3	9
$-2d'^5$	3	1	5	3
$-2f'^5$	3	2	4	4
$-1g'^5$	1	4	0	6
$-(1g'^3)$	1	1	0	6
$-3d'^5$	.....	.....	4	8

E.P. Multiplet	3	2	1	0
	Iron—*Continued*			
(1.54)				
$1f^3-1d'^3$	.....	.....	1	4
$-(1g'^5)$	.....	1	2	5
$-1g'^3$	.....	2	2	2
$-2f'^3$	.....	3	2	2
$-2d'^3$	1	2	1	2
(2.19)				
$1p^5-3d'^5$	.....	.....	1	8
(2.46)				
$1d'^7-md^7$	.....	.....	2	10
$-nd^7$	.....	.....	1	(11)†
(2.84)				
$1f'^7-md^7$	.....	1	2	12
(2.96)				
$1p'^7-md^7$	.....	.....	1	8
(3.25)				
$1d'^5-md^5$	.....	.....	1	11
(3.38)				
$1f'^5-md^5$	.....	.....	1	11

* Lines to the violet of λ 3022 excluded.

† Assumed—multiplet not complete.

from those of the widening of many emission lines in laboratory spectra. The evidence for this is simply that the lines which are widened in the laboratory are more often than not sharp in the sun, and vice versa. For example, the lines of the diffuse singlet series of *Ca*, notoriously hazy in the artificial sources, are quite sharp in the sun. Again, the iron lines λλ 5424.06, 5415.19, and 5404.13, which are very hazy in the arc, are sharp in the sun, while λ 5269.54 (for example) is winged in the sun, though distinctly sharp in the arc.

Many other instances on both sides might be cited. This is not at all surprising, for the widening in the laboratory almost always happens under conditions which indicate a density effect (i.e., the influence of neighboring atoms charged or neutral) while there is good reason to believe that the density of the reversing layer is very low.[1]

The widening of the hydrogen lines in the sun, however, may be largely due to the influence of neighboring molecules, since *H* is unusually sensitive to the Stark effect. It is also, of course, peculiarly sensitive to Doppler widening.

For the remaining winged solar lines it appears to be sufficient to assume that the widening depends mainly, if not entirely, upon the number of atoms in the solar atmosphere which are in a condition, at any given instant, to deal with radiation of the given frequency. By no means all the atoms of the element in question will be in such a condition. They must be, in the first place, in the right degree of ionization and in the appropriate energy-state, the fraction of the whole which are so being governed probably by thermodynamic considerations. Furthermore, not all the atoms which are in a state to absorb a given line will actually do so; many of them will be busy (so to speak) with other lines arising from the same energy-level. The relative numbers in this case are probably determined by quantum conditions, which have very recently been elucidated so far as the components of a given multiplet are concerned, but remain to be worked out for different multiplets arising from the same term. It is already clear, however, that the more probable transitions correspond to the stronger lines, as observed in the laboratory; that transitions to the lower terms of a series are more probable than to the higher terms, and so on.

We should therefore expect that, in the case of an abundant element, the multiplets arising from the lowest energy-levels should show all but their weakest lines widened, and their strongest lines greatly winged, while, on passing to sufficiently high levels only the strongest lines should be widened, and that but slightly. This is exactly what is observed among the solar iron lines, while the ex-

[1] Fowler and Milne, *Monthly Notices of the Royal Astronomical Society*, 84, 499, 1924; Russell and Stewart, *Astrophysical Journal*, 59, 197, 1924.

pected falling off in widening among the later members of a series is beautifully shown in magnesium.

Further evidence is found in the case of calcium and titanium. Both thermodynamic calculations and the behavior of the lines in sun-spots show that, in the reversing layer, most of the atoms of these elements must be singly ionized; and their enhanced lines are much more widened in the sun than are their arc lines.

If the proposition is accepted that the line-widening is roughly proportional to abundance, interesting conclusions follow concerning the relative proportions of various elements in the sun's atmosphere. Roughly speaking, the abundant elements should be those which appear in Table II; but the order in which they there

TABLE IV

	Planet	Crust		Planet	Crust
Fe	39.76	4.68	*Co*	0.23	0.003
O	27.71	49.19	*Cr*	.20	0.06
Si	14.53	25.71	*K*	.14	2.38
Mg	8.69	1.94	*P*	.11	0.14
Ni	3.16	0.03	*Mn*	.07	.11
Ca	2.52	3.37	*C*	.04	.14
Al	1.79	7.50	*Ti*	0.02	.65
S	0.64	0.09	*H*	.....	.87
Na	0.39	2.61	*Cl*	.....	0.23

stand is much affected by the limitations of the observable solar spectrum. Silicon, for example, has but a single accessible strong line, all the strongest, including the ultimate and penultimate lines, lying beyond λ 3000. The ultimate lines of *Mg* and Mg^+ are also unobservable. Sodium, on the other hand, must be very heavily, and potassium almost completely, ionized above the photosphere.

Bearing these things in mind, it is of interest to compare Table II with the estimates of Clarke and Washington regarding the composition of the earth's crust[1] (including ocean and atmosphere) and of the planet as a whole,[2] which are given in Table IV (in percentages by weight).

The recent estimate of the composition of the earth as a whole shows a striking parallelism with the indications of the solar spec-

[1] *Proceedings of the National Academy of Sciences*, **8**, 114, 1922.

[2] *American Journal of Science*, **9**, 351, 1925.

trum regarding that of the sun's atmosphere. The importance of iron, magnesium, nickel, and calcium on both lists is noteworthy. Aluminium is considerably handicapped in the sun, silicon very heavily, and oxygen hopelessly—its ultimate lines lying in the Schumann region, and the well-known group in the red having so high an excitation potential (9.11 volts) that its faintness in the sun appears inevitable. The same is true of sulphur. Sodium and titanium show a strength in the sun which would harmonize better with their estimated abundance in the earth's crust than in the planet as a whole.

The general agreement, however, is such as to add further support to the conclusion that the widening of solar lines is primarily a result of the abundance of the atoms concerned. It is of course not suggested that other factors, peculiar to the line considered, may not come into play, but only that abundance is the major factor, and capable of accounting qualitatively for the facts so far observed. (Rough formulae connecting line-width with the number of active atoms have been given by J. Q. Stewart.)[1]

Exception must again be made of the hydrogen lines. Their strength, even more than their width (which might perhaps be accounted for by the Doppler and Stark effects), indicates an abundance of hydrogen atoms in the two-quantum state which is very hard to reconcile with present thermodynamic theory. Rosseland's recent proof[2] that hydrogen may be greatly concentrated toward the surface of a star may help to clear away the difficulty.

There is evidently great promise in quantitative work, both in the measurement of the degree of widening of solar lines and in their theoretical discussion, and it is largely in the hope of stimulating such work that the present reconnaissance of the field has been undertaken. It may be mentioned in closing that Miss Moore is responsible for the first part of the paper, and Mr. Russell for the theoretical discussion.

PRINCETON UNIVERSITY OBSERVATORY
September 28, 1925

[1] *Astrophysical Journal*, **59**, 30, 1924.

[2] *Monthly Notices of the Royal Astronomical Society*, **85**, 541, 1925.

NOTES ON LINES IN THE SPECTRA OF RED STARS[1]

By PAUL W. MERRILL

ABSTRACT

Lines of various elements in stellar spectra of classes M, N, and S.—The behavior of lines of *H, He, Li, Na, K, Mg, Ca, Sr, Ba, Al, Ti, Zr, V, Cr, Mn,* and *Fe* is described, especial attention being paid to the series classification of the lines and their relative intensities in the various spectral types.

The rising interest in the chemical and physical problems of stellar atmospheres makes desirable the compilation of detailed data concerning stellar spectra in a form adapted for convenient reference. Professor Russell has prepared the way for this by publishing a classified list of spectral lines of astrophysical interest.[2] The purpose of the present paper is to describe lines of various chemical elements in the spectra of stars near the low-temperature end of the stellar sequence. The data refer chiefly to giant variable stars of classes M, N, and S.

A few introductory remarks will aid in the interpretation of the data which follow. In the first place, it is to be remembered that the dispersion with which spectrograms of red stars have been obtained is very small compared to that used in laboratory and solar research. This means that very accurate determinations of wave-length cannot be made; that faint lines cannot be observed; and that difficulties caused by the blending of neighboring lines will be numerous and troublesome.

The spectral region which has been extensively observed is limited to a few hundred angstrom units. The region most adequately studied lies between λ 4100 and λ 4500. At shorter wave-lengths the rapid drop in the spectral intensity-curve results in underexposed spectrograms, which rarely extend to the calcium line K (λ 3933). At wave-lengths longer than λ 4500, and especially beyond λ 4900, the observations are not so numerous, the prismatic dispersion is decidedly smaller, and the strong bands greatly inter-

[1] *Contributions from the Mount Wilson Observatory*, No. 306.

[2] *Mt. Wilson Contr.*, No. 286; *Astrophysical Journal*, **61**, 223, 1925.

fere with the recognition and identification of lines. In attempting the present compilation the writer has been much impressed by the incompleteness of the data, especially outside the spectral limits λ 4100 to λ 4900. This is caused partly by inherent difficulties and partly by lack of adequate observations. A comparison of our knowledge of lines in stellar spectra with Russell's lists and other laboratory data will suggest numerous definite problems, many of which would readily yield to systematic observation.

TABLE I

CHARACTERISTIC INTENSITIES OF BRIGHT HYDROGEN LINES IN VARIABLE STARS AT MAXIMUM LIGHT

	Class M	Class S	Class N
Hα 6563	2	15	10
Hβ 4861	2	12	10
Hγ 4340	20	5	5
Hδ 4102	30	3	2
Hε 3970	2	.....	.....
Hζ 3889	15	1	Trace
Hη 3835	8	.....	.....
Hθ 3798	5	.....	.....
Hι 3771	7	.....	.....
Hκ 3750	1	.....	.....

The very valuable discussions of lines in spectra of class N by Hale, Ellerman, and Parkhurst,[1] and by Shane,[2] have been freely used in preparing the present paper. Through the kindness of Mr. Joy, his extensive investigation of *o* Ceti has been available for reference prior to its publication.

Hydrogen.—The Balmer lines are bright in all long-period variables of classes M and S, and in a few variables of class N. In all these classes the bright hydrogen lines have their greatest intensities near the time of maximum light and are weak or absent near minimum. Typical intensities at maximum are given in Table I.[3]

The anomalously low intensities of bright *Hα*, *Hβ*, and *Hε* in Me stars are very striking and present important physical problems.[4]

[1] *Publications of the Yerkes Observatory*, **2**, 253, 1903.

[2] *Lick Observatory Bulletins*, **10**, 79, 1920.

[3] For more detailed data, see the following references: Class Me, *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 185, 1921. Class Se, *Mt. Wilson Contr.*, No. 252; *Astrophysical Journal*, **56**, 457, 1922. Class Ne, *Lick Observatory Bulletins*, **10**, 79, 1920.

[4] *Physical Review*, **25**, 717, 1925.

It appears to be true in general that, the cooler the star, the greater is the departure of the relative intensities of the Balmer lines from those ordinarily observed in the laboratory.

The Balmer lines are frequently present in absorption in irregular variables and non-variable red stars. They are fairly strong in S-type spectra and high-luminosity stars of classes Mo–M3. They are weak in advanced M-types and in all red dwarf stars. In class N $H\gamma$ and $H\delta$ have been observed as dark lines,[1] but not $H\alpha$ and $H\beta$[1, 2]

Lines of the secondary spectrum of hydrogen have not been observed in stellar spectra.

Helium.—Lines of helium are not ordinarily found in the spectra of red stars. They appear, however, in R Aquarii at minimum brightness, but the spectrum at this time strongly resembles that of a planetary nebula, and its physical relationship to the M7e spectrum of the variable star is not understood.

Lithium.—No lines have been identified with certainty. The strong lines are not in favorable portions of the spectrum, but the intensities cannot be high.

Sodium.—The dark D lines are strong in most red stars, being outstanding features in classes N and S. In class M, however, they are not nearly so strong; in the advanced types M6e–M8e they are, in fact, surprisingly inconspicuous. Thus, in the coolest stars, these lines have but a small fraction of the intensity they exhibit at somewhat higher temperatures. As they are ultimate lines of neutral sodium, this behavior is unexpected and suggests that at the lowest-known stellar temperatures the supply of free sodium atoms in the reversing layer becomes nearly exhausted, perhaps by the formation of compounds.

Potassium.—The pair λλ 4044, 4047, $1s^2 - 2p^2$, is weak in types Mo to M4, but strengthens rapidly toward the end of the M sequence. This behavior is to be expected of lines in the principal series, and is at variance with the anomalous weakness of the D lines of sodium in the coolest stars. The potassium pair merits further study as its region has not been adequately observed in N and S spectra. Among M-type spectra it appears not always to have the same in-

[1] Hale, Ellerman, and Parkhurst, *op. cit.*, p. 373, 1903.

[2] Shane, *Lick Observatory Bulletins*, **10**, 81, 1920.

tensity in stars of a given subdivision, and perhaps varies with the light-phase.

Magnesium.—The low-temperature line λ 4571.11, $1S-1p^3$, is present in absorption in long-period variables of classes M and S near maximum light, changing to emission toward minimum.[1] At minimum it may become the strongest feature in the blue-violet portion of the spectrum. It is present as a weak absorption line in N-type spectra.

The green triplet λλ 5183.67, 5172.70, 5167.38, $1p^3-1s^3$, is described as "prominent" in absorption in class N by Hale, Ellerman, and Parkhurst. It is probably present in class S. It is difficult to observe in class M, at least with low dispersion, on account of the strong titanium oxide band having its head at λ 5167, but has been measured in the spectrum of α Herculis.

The first member of the diffuse triplet series $1p^3-2d^3$ consists of lines at λλ 3838.29, 3832.31, and 3829.36. Faint emission lines were measured in these positions on a single-prism spectrogram of R Leonis taken at Mount Wilson on February 26, 1921, forty-seven days after maximum light. There was only a trace of bright λ 4571 at this time, but a month later it was strongly developed. Unfortunately nothing can be said as to the intensities of the ultra-violet triplet on the later date. When properly observed, the relative behavior of these magnesium lines should yield valuable information concerning temperatures in the atmospheres of these stars.

Calcium.—The ultimate singlet λ 4226.73 is very strong and broad in absorption in red stars, and, in contrast to the behavior of the D lines of sodium, seems to grow more intense with decreasing temperature. λ 6572.78, $1S-1p^3$, if present at all, is weak in all classes of red stars.

The group given by Russell under excitation potential 1.88 is well represented in advanced M-types and probably also in N- and S-types. All the class I lines in this group seem to be present in M-type variables, although they are not strong, and a few are doubtful because nearly coincident with low-temperature lines of other elements.

[1] *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 185, 1921. *Publications of the Astronomical Society of the Pacific*, **33**, 206, 1921.

The ionized lines, Fraunhofer H and K, lie so far in the violet that abnormally long exposures are required to show them in the spectra of red stars. They are conspicuous broad dark lines on spectrograms having the proper density in their region and are doubtless present in all or nearly all red stars. Although probably somewhat less intense than in class Ko, they are nevertheless surprisingly strong even in the cooler red stars such as χ Cygni. More extensive observations of these lines are very desirable to indicate their intensities in different stellar types and, in variable stars, their changes with phase.

Strontium.—The strong low-temperature line λ 4607.34, $1S-1P$, is well marked in red stars. In class M it appears, in general, to grow stronger with advancing type. The line λ 4832.08 has been measured in α Herculis, class M5.

The ionized lines $\lambda\lambda$ 4077.71 and 4215.52 are strong in class N, very strong in class S, and have widely different intensities in various M-type stars. While there is a strong tendency for these lines to grow weaker from M3 to M8, absolute magnitude, or some other factor, introduces considerable variations. These lines are very much stronger in χ Cygni, for example, than in certain other stars of the same type. In a few Me variables they have been observed to grow less intense after maximum light.

Barium.—The strong low-temperature line λ 5535.53, $1S-1P$, is weak or absent in all three types of red stars.

The ionized line λ 4554.04 is weak in class M, strong in class N, and very strong in class S, being, in fact, one of the distinctive features of class S. Its behavior is followed by that of the weaker line λ 4934.10.

Aluminum.—The pair $\lambda\lambda$ 3944.03 and 3961.54, $1p^2-1s^2$, is strong in Me variables. Mr. Joy informs me that in o Ceti it strengthens markedly toward minimum. Its intensity in classes N and S is not known.

Titanium.—The lines $\lambda\lambda$ 3981.77, 3989.77, and 3998.65 of the 0.0 energy-level of the neutral atom are strong in all subdivisions of class M, probably increasing in intensity with advancing type. Several others of the stronger lines arising from the 0.0 level have also been observed. The lines near λ 4500 from the level 0.82 are

fairly strong in the early subdivisions, but decrease toward M8. This group of lines is present in N and S spectra also.

Zirconium.—The following strong zirconium lines are present in class S:

4535.75 blend with *Ti*	4710.08
4542.22?	4772.33
4575.52	4815.06 }
4633.99	4815.62 }
4687.81 }	4828.06
4688.45 }	

They appear to be absent or much weaker in class M. This probably applies to class N also.

Vanadium.—Numerous lines from the two lowest levels are present in classes M, S, and N, although many are of moderate intensity. In class Me, which has been more extensively observed, these lower-level lines are represented with remarkable completeness.

Chromium.—The well-known triplet λλ 4254, 4274, 4289, $1s^7 - 1p^7$, is strong in all classes of red stars. In class M it shows a marked increase of intensity with advancing type. The green triplet near λ 5206 is present in all three classes. In class M nearly all the stronger lines in the 0.94 and 0.98 levels which lie in accessible parts of the spectrum have been observed, and λ 5328.34, the strongest line in the 2.88 group, has been measured in α Herculis.

Manganese.—The well-known low-temperature triplet near λ 4030, $1s^6 - 1p^6$, is very strong in M-type stars, increasing in intensity with advancing type. It is present in class S with approximately the same intensity as in the earlier subdivisions of class M. In Me variables, λ 4030 has been observed in emission during the phase of decreasing light. It appears first as a bright edge on the violet side of the dark line.

The multiplet $1d^6 - 1d'^6$ is represented by weak lines in class Me. They appear not to strengthen with advancing type to the same extent as the λ 4030 triplet. This might be expected because the level to which they belong is 2.16 volts above that for the triplet.

The line λ 4823.52 has been measured in all three classes of red stars.

Iron.—Comparing the iron-arc spectrum with lines in class M, we find a fragmentary and irregular correspondence, but if we use instead a list of King's low-temperature lines, or the lines originating from the 0.0 and 0.94 levels as compiled by Russell, the agreement is quite complete and satisfactory. The strong lines of level 1.54 are present with relatively low intensities, and lines of higher levels appear to be absent. The lines of the 1.54 level grow weaker with advancing type. Classes N and S resemble class M, except that the lines of level 1.54 are relatively stronger. Toward minimum of long-period variables of class Me the extreme low-temperature lines appear in emission. At an earlier phase after maximum, strong emission lines appear in the positions of the iron lines λλ 4202 and 4308. These are believed by several observers to arise from iron atoms, but some doubt as to the identification will probably remain until these lines can be shown by laboratory experiments to be outstanding under certain conditions. It is possible, however, that continued observations of stellar spectra may settle the matter by showing the presence in emission of related iron lines.

A remarkable fact, but one which seems well attested, is the presence of a few of the strongest enhanced lines of iron in Se variables,[1] in *o* Ceti,[2] and in certain other M stars.[3]

MOUNT WILSON OBSERVATORY
September 1925

[1] *Mt. Wilson Contr.*, No. 252; *Astrophysical Journal*, **56**, 457, 1922.

[2] Unpublished data by Mr. Joy.

[3] *Publications of the Astronomical Society of the Pacific*, **34**, 175, 1922.

THE MAGNESIUM HYDRIDE BAND SPECTRUM

By WILLIAM W. WATSON and PHILIP RUDNICK

ABSTRACT

General structure and series assignments.—New wave-length determinations of the lines in the λλ 5622, 5211, and 4845 bands have been made, and the several *P*, *Q*, and *R* branches, all terms of which are doublets, assigned. A correlation between abnormal separations and unequal intensities in the doublets near the origins of the bands is noted.

Term structure.—The various combination relations are tested, and it is shown that the combination involving the *Q* branch does not hold in any one band. Rotational and vibrational quantum assignments are made, and the form of the approximate final term is found to be given empirically by $\Delta f(m) = 12.0m - 0.06m^2$.

Isotope bands.—The predicted isotope effect for a diatomic *MgH* molecule exists in all the bands, namely, the occurrence of two faint companions to each band line, and with the displacements as given by the theory.

The carrier of the bands.—Reasons for considering the carrier to be *MgH* are presented. A possible identification of the electronic frequency with an electron transition in the *Mg* atom is made. The moment of inertia of a dipolar carrier is computed to be 4.6×10^{-40} gm. cm², and the internuclear distance 1.7×10^{-8} cm.

INTRODUCTION

The emission bands with principal heads at λλ 5622, 5211, and 4845, attributed with considerable certainty to a compound of magnesium and hydrogen, were investigated in some detail by A. Fowler,[1] whose principal object, aside from a purely empirical description of the spectrum, was the identification of the chief lines with absorption lines in the spectra of sun-spots. Heurlinger[2] in his comprehensive study of the fine structure of representative classes of bands considered the magnesium hydride spectrum, making use of Fowler's wave-lengths. But since the applications of the quantum theory to the interpretation of band structure were then meager and unsatisfactory compared to the present theory of band spectrum, a further investigation of the fine structure of these bands was undertaken. The purpose of this paper is to present the new data in so far as they bear upon the questions of term structure, isotope bands, and the nature of the carrier of the spectrum.

It was found necessary to replace Fowler's wave-lengths with new experimental data. The source employed was a modification of that used by Fowler, an arc operated between magnesium electrodes

[1] *Philosophical Transactions of the Royal Society of London*, A, 209, 447, 1909.

[2] Dissertation, Lund, 1918.

in an atmosphere of hydrogen. Photographs of the region from λ 5700 to λ 4300 were taken in the second order of a concave grating of focal length 21 feet; the dispersion obtained was 1.3 A per millimeter, as compared with $5\frac{1}{2}$ and $2\frac{3}{4}$ A per millimeter secured by Fowler. All wave-lengths were measured with reference to the secondary iron standards, and reductions to vacuum frequencies were made with the aid of the tables of the United States Bureau of

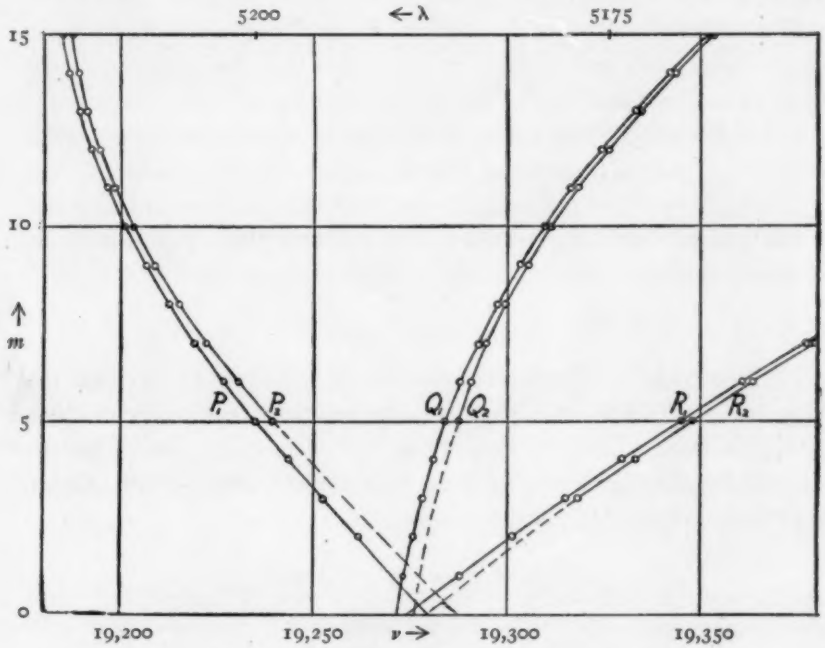


FIG. 1.—λ 5211 band of MgH_2. $n=n=0$

Standards. Agreement with Fowler's measurements was found to be quite close, except that many more lines were resolved on the writers' plates.

The first two sets of *P*, *Q*, and *R* branches in both the λ 5211 and the λ 5622 groups, together with one set of branches in the λ 4845 group, have been found, and assignments of rotational and vibrational quantum numbers have been made with the aid of the combination relations discussed in the next section. As indicated in Figure 1, all of the branches are composed of doublets whose frequency

separations are quite small and nearly constant for all but the lowest values of m. The normal separations lie between 1.5 and 2.0 cm^{-1}, but increase for the lowest values of m to 5.0 cm^{-1} and more. For large values of m, the doublet components are of equal intensity, but as the branches approach the axis $m=0$, the components of shorter wave-length become much fainter than their companions. Indeed it is only the components of long wave-length which can be traced all the way to the axis.

Furthermore, a careful inspection of the P-branch doublets leads one to the definite and suggestive conclusion that the departure from equal intensities and the departure from the normal frequency separation begin at the same point, and increase together as m decreases; so that one might expect the two departures to be associated with some property of the radiating molecule. Observations on this point in the Q and R branches cannot be made very definitely, but it seems probable that the relation holds in their cases also.

TERM STRUCTURE

It is essential to the point of view of the quantum theory that the frequencies of band lines shall be expressible as differences of frequency terms, which we may associate with energy-levels in the radiating molecule. The following is the usual term representation of corresponding P, Q, and R lines:

$$P(m)=F(n', m-1)-f(n, m)$$
$$Q(m)=F(n', m)-f(n, m)$$
$$R(m)=F(n', m+1)-f(n, m)\ .$$

Keeping n' and n fixed and allowing m a range of integral values, 1, 2, 3, ... , gives the lines of the (n', n) band. F and f refer to energy-levels which represent, respectively, initial and final states of the molecule in the process of radiation, usually thought of, in the case of the visible and ultra-violet bands, as involving a change in electronic configuration. This transition supplies the major part of the energy of radiation, and corresponds to the ν_0 frequency of the band system. n' and n are vibrational quantum numbers, and may be equal or differ by one or more units. The integer m, as used here,

is not necessarily the rotational quantum number, but may differ from it by a small constant, usually $\pm\frac{1}{4}$ or $\pm\frac{1}{2}$, depending upon the specific term formula and molecular model used.

A necessary and sufficient condition that a given set of P, Q, and R frequencies shall be representable as differences in the foregoing manner is

$$R(m)-Q(m+1)=Q(m)-P(m+1)\ . \qquad (A)$$

If this condition is satisfied, the F and f may be obtained, except for an additive constant, from the relations

$$R(m)-Q(m)=Q(m+1)-P(m+1)=F(n',\ m+1)-F(n',\ m)=\Delta F(n',\ m)$$
$$R(m)-Q(m+1)=Q(m)-P(m+1)=f(n,\ m+1)-f(n,\ m)=\Delta f(n,\ m)\ .$$

A further hypothesis very generally made is that the same set of F or f terms may appear in more than one band, as, for example:

$$P(n',\ n_1;\ m)=F(n',\ m-1)-f(n_1,\ m) \qquad P(n',\ n_2;\ m)=F(n',\ m-1)-f(n_2,\ m)$$
$$Q(n',\ n_1;\ m)=F(n',\ m) \quad -f(n_1,\ m) \qquad Q(n',\ n_2;\ m)=F(n',\ m) \quad -f(n_2,\ m)$$
$$R(n',\ n_1;\ m)=F(n',\ m+1)-f(n_1,\ m) \qquad R(n',\ n_2;\ m)=F(n',\ m+1)-f(n_2,\ m)$$

This term formulation may be tested similarly by the condition

$$Q(n',\ n_2;\ m)-P(n',\ n_2;\ m)=Q(n',\ n_1;\ m)-P(n',\ n_1;\ m)\ , \qquad (B)$$

and by similar relations involving the differences $Q-R$ and $P-R$. As a matter of fact, the parameter n in the F and f terms is used to account for the twofold multiplicity of partial bands in a band system. In such a scheme, groups of bands will occur such that for corresponding P lines, for example, the following term representations will be used:

$$P(n_1,\ n_1;\ m)=F(n_1,\ m-1)-f(n_1,\ m) \qquad P(n_1,\ n_2;\ m)=F(n_1,\ m-1)-f(n_2,\ m)$$
$$P(n_2,\ n_1;\ m)=F(n_2,\ m-1)-f(n_1,\ m) \qquad P(n_2,\ n_2;\ m)=F(n_2,\ m-1)-f(n_2,\ m)$$

and similarly for Q and R lines. In order to justify this representation the following relation must be satisfied:

$$P(n_1,\ n_1;\ m)-P(n_2,\ n_1;\ m)=P(n_1,\ n_2;\ m)-P(n_2,\ n_2;\ m)\ , \qquad (C)$$

and of course the analogous relations for Q and R lines likewise. Equation (C) is a condition for vibrational term structure in just the same way that (A) is for the rotational case.

Criteria of term structure of the form of (A), (B), or (C) are said to be forms of the combination principle. They constitute a critical test of the present theory of the band spectra in its simple form, since to account for combination defects one must introduce additional energy-levels for which there is at present no definite concrete interpretation.

A trial of relation (A) as applied to the doublet components of longer wave-length in the first partial bands of the groups λ 5211 and λ 5622 is given in Table I. It is readily seen that, while near the band origin the departures from the law are random and must be attributed to experimental error, for higher values of m the $P-Q$ differences grow systematically smaller than the corresponding $Q-R$ differences. The same situation undoubtedly occurs in all the other partial bands of the system, though there the series are not known to so many terms, and the departures are consequently not so definitely seen.

In making the assignment of vibrational quantum numbers, the first partial band of the group λ 5211 is selected as the (0, 0) band because of its predominating intensity and its symmetric position between the two fainter band groups. It then appears from combination relations of the type (B), as indicated in Table II, that the first partial bands of the groups λ 5211 and λ 5622 have common initial terms, and that the second band at λ 5211 and the first at λ 5622 have common final terms. The suggested vibrational assignment for the second band at λ 5211 is then (1, 1) and that for the first at λ 5622 is (0, 1). Further investigations of the same sort show that the general assignments $(n, n+1)$, (n, n), and $(n+1, n)$, respectively, for the groups at λ 5622, λ 5211, and λ 4845, are consistent with all applicable combination relations of type (B). It is to be noted that these relations are satisfied within the experimental uncertainty of the wave-length determinations for all values of m. There is no evidence of a combination defect.

Table III gives a comparison of differences between lines of the (1, 0), (0, 0), (1, 1), and (0, 1) bands of the type of relation (C).

Again the departures from equality are random, and there is no indication of a combination defect.

TABLE I

m	λ 5211 Band			λ 5622 Band		
	$R(m)-Q(m+1)=\Delta f(m)$	$Q(m)-P(m+1)=\Delta f(m)$	Mean Departure	$R(m)-Q(m+1)$	$Q(m)-P(m+1)$	Mean Departure
1.....	11.97	11.65	.....	.....	.....	.....
2.....	22.98	23.05	.....	22.66	.....	.....
3.....	34.12	23.34	.....	33.23	33.44	.....
4.....	45.88	46.04	.....	44.36	44.61	.....
5.....	57.13	57.02	−0.03	55.28	55.43	−0.20
6.....	68.43	68.54	.....	66.35	66.47	.....
7.....	79.78	79.89	.....	77.30	77.41	.....
8.....	90.90	91.06	.....	87.99	88.07	.....
9.....	102.10	102.27	.....	98.87	99.23	.....
10.....	13.46	13.36	− .09	109.74	109.68	− .12
11.....	124.50	124.22	.....	120.25	120.50	.....
12.....	35.35	35.20	.....	30.97	30.95	.....
13.....	46.29	46.27	.....	41.56	41.42	.....
14.....	57.06	57.08	.....	51.99	51.64	.....
15.....	67.74	67.21	+ .19	62.14	61.74	+ .13
16.....	178.22	177.73	.....	172.25	171.77	.....
17.....	88.66	88.00	.....	82.33	81.85	.....
18.....	98.83	98.05	.....	92.09	91.70	.....
19.....	208.98	208.18	.....	201.78	201.31	.....
20.....	18.84	17.99	+ .72	11.39	10.90	+0.46
21.....	228.66	227.81	.....	.....	.....	.....
22.....	38.21	37.40	.....	.....	.....	.....
23.....	47.61	46.73	.....	.....	.....	.....
24.....	56.88	56.10	.....	.....	.....	.....
25.....	65.95	65.13	+. 83	.....	.....	.....
26.....	274.91	273.92	.....	.....	.....	.....
27.....	83.49	82.61	.....	.....	.....	.....
28.....	92.14	91.21	.....	.....	.....	.....
29.....	300.15	99.34	.....	.....	.....	.....
30.....	308.15	307.55	+ .84	.....	.....	.....
31.....	316.20	315.33	.....	.....	.....	.....
32.....	23.54	22.41	.....	.....	.....	.....
33.....	30.96	30.15	+0.90	.....	.....	.....

Combination defects similar to those brought out in Table I appear in some of the $(C+H)$ bands[1] and in the OH bands.[2] Birge[3]

[1] Kratzer, *Zeitschrift für Physik*, **23**, 298, 1924.

[2] Watson, *Astrophysical Journal*, **60**, 145, 1924.

[3] *Physical Review*, **25**, 240, 1925.

and Dieke[1] have considered the latter case in some detail, and conclude that the Q branches have their final states in a series of dis-

TABLE II

m	$Q(0,0;m)-P(0,0;m)=\Delta F(0,m)$	$Q(0,1;m)-P(0,1;m)=\Delta F(0,m)$	$Q(0,1;m)-P(0,1;m+1)=\Delta f(1,m)$	$Q(1,1;m)-P(1,1;m+1)=\Delta f(1,m)$
1	.....	.....	.....	.....
2	14.22	.....	.....	.....
3	25.37	25.42	33.44	.....
4	37.24	37.32	44.61	.....
5	48.97	49.08	55.43	.....
6	60.95	61.03	66.47	.....
7	73.08	73.11	77.41	77.19
8	84.87	85.03	88.07	88.14
9	96.92	96.97	99.23	98.95
10	108.71	108.69	109.68	109.73
11	120.35	120.50	120.50	120.38
12	32.04	32.34	30.05	30.89
13	43.78	43.89	41.42	41.29
14	55.35	55.24	51.64	51.57
15	66.92	66.48	61.74	61.76
16	177.73	177.72	171.77	172.03
17	88.88	88.88	81.85	81.78
18	99.92	99.97	91.70	91.68
19	210.56	210.76	201.31	201.53
20	21.32	21.43	10.90	10.98
21	231.84	232.02	.....	.....
22	42.27	42.33	.....	.....
23	52.47	52.55	.....	.....
24	62.52	62.50	.....	.....
25	72.34	72.25	.....	.....
26	282.05	281.94	.....	.....
27	91.37	91.23	.....	.....
28	300.85	300.64	.....	.....
29	09.69	10.14	.....	.....
30	18.76	18.54	.....	.....

placed energy-levels $f'(m)$. This may be shown in the case of the OH bands by noting first that

$$f_P(m)=f_R(m),$$

since appropriate $P-R$ differences are equal for all values of m, but that

$$2\Delta f(m)=f(m+2)-f(m)=R(m)-P(m+2)$$

[1] *Proceedings of the Amsterdam Academy*, **28**, 174, 1925.

differs by progressively larger amounts from

$$2\Delta f'(m)=f'(m+2)-f'(m)=R(m+1)-Q(m+2)+Q(m)-P(m+1)$$

as m increases. Many satellites which are observed in the *OH* band λ 3064 may be attributable to these displaced levels. The question holds interesting possibilities, but as yet little has been done toward answering it.

TABLE III

m	$P(1,0;m)-P(0,0;m)$	$P(1,1;m)-P(0,1;m)$
8	1526.63	1526.62
9	1523.95	1524.27
10	1520.63	1520.82
11	1517.60	1517.51
12	1513.27	1513.61
13	1509.50	1509.27
14	1505.12	1504.84
15	1500.40	1499.80
16	1494.55	1494.50
17	1488.86	1488.55
18	1482.78	1482.68
19	1475.91	1476.00
20	1469.10	1468.82
21	1461.73	1461.64
22	1454.08	1453.84
23	1445.84	1445.80
24	1437.28	1437.07
25	1428.39	1428.00

In view of the general similarity betwen the *MgH* and *OH* bands, it seems a reasonable possibility that the *MgH* bands, too, owe their combination defect to the existence of displaced Q levels, but it has not been found possible to verify this supposition because of the smallness of the defect. It is to be noted that both band systems have the same characteristic spreading of the doublets at the band origin and the same combination defect, both more pronounced in the case of the *OH* bands. The question of the existence of corresponding satellite lines in the *MgH* bands is a difficult one to investigate because of the presence of the many faint lines of the isotope bands.

Referring to Table I, one may take as an approximation to $\Delta f(m)$ the differences $Q(m)-P(m+1)$ appearing in the third column.

These may be represented with sufficient accuracy by the empirical relation:

$$\Delta f(m) = Q(m) - P(m+1) = 12.0m - 0.06m^2 . \qquad (D)$$

This formula is of interest in its contrast with the form given to $\Delta f(m)$ by the Kramers and Pauli term,[1]

$$f(m) = b(\sqrt{m^2-\sigma^2} - \epsilon)^2 ,$$

whose corresponding value for $\Delta f(m)$ is[2]

$$\Delta f(m) = b\left[2(m - \epsilon + \tfrac{1}{2}) - \frac{\epsilon\sigma^2}{m(m+1)}\right] .$$

In this formula the departure from linearity occurs for small values of m, while in the empirical relation (D) the departure is for large values of m. In view of this difference it seems unlikely that the Kramers and Pauli term and molecular model could be used successfully in the representation and interpretation of this band system.

ISOTOPE BANDS

The theory of the isotope effect in band spectra has recently been summarized and developed by Mulliken.[3] His discussion for the first-order effect follows in brief.

Consider a set of dipolar molecules in which an element appears in its various isotopic modifications of masses M_i. μ_i is defined in the usual manner,

$$\frac{1}{\mu_i} = \frac{1}{M_i} + \frac{1}{M'} ,$$

where M' is the mass of the other atom in the dipole. Now the energy of a rotating and vibrating dipolar molecule, capable of radiating a band spectrum of the electronic type, may be written as the

[1] *Zeitschrift für Physik*, **13**, 351, 1923.

[2] Kratzer, *loc. cit.*

[3] *Physical Review*, **25**, 119, 1925.

sum of three terms, in which the coefficients may be shown on very general grounds to depend upon μ only in the following way:

$$E=h(\nu_e+\nu_n+\nu_m)=h\nu_e+h\left(\frac{a}{\mu^{\frac{1}{2}}}n-\frac{b}{\mu}n^2+\ \ldots\ .\right)$$
$$+h\left(\frac{A}{\mu^{\frac{1}{2}}}m+\frac{B}{\mu}m^2+\ \ldots\ .\right).$$

ν_e is independent of mass to about the same degree of approximation as are the terms of series spectra.

Observing that the linear term in n and the quadratic term in m are the prominent ones in the vibrational and rotational parts of the energy expression, respectively, one may write as an approximation,

$$\frac{\nu_n^{(2)}}{\nu_n^{(1)}}=\frac{\sqrt{\frac{1}{\mu_2}}}{\sqrt{\frac{1}{\mu_1}}} \text{ and } \frac{\nu_m^{(2)}}{\nu_m^{(1)}}=\frac{\frac{1}{\mu_2}}{\frac{1}{\mu_1}},$$

or, setting

$$\rho=\frac{\sqrt{\frac{1}{\mu_2}}}{\sqrt{\frac{1}{\mu_1}}},$$
$$\nu_n^{(2)}-\nu_n^{(1)}=(\rho-1)\nu_n^{(1)}$$
$$\nu_m^{(2)}-\nu_m^{(1)}=(\rho^2-1)\nu_m^{(1)}.$$

This gives the frequency difference between corresponding lines associated with isotopes of masses M_1 and M_2. It may be noted that the coefficient in the displacement formula for the rotational case is very nearly twice that for the vibrational case.

Taking magnesium to have isotopes of masses 24, 25, and 26, of relative abundance about 7:1:1,[1] one expects the spectrum of a magnesium-hydrogen dipole to consist of a set of strong lines associated with Mg_{24}, accompanied by two fainter systems whose displacements are given by

$$\rho_1^2-1=\frac{-1}{625} \text{ and } \rho_2^2-1=\frac{-1}{325}.$$

[1] Dempster, *Physical Review*, **18**, 415, 1921.

For all other than the (0, 0) band, the displacements should be compounded of both rotational and vibrational parts. In the (0, 0) band the only effect of readily observable magnitude should be the pairs of rotationally shifted lines to the red of their stronger companions.

An inspection of photographs of this band gives unmistakable indication that the predicted effect exists. There appear, on the low-frequency side of all the prominent series lines, pairs of faint com-

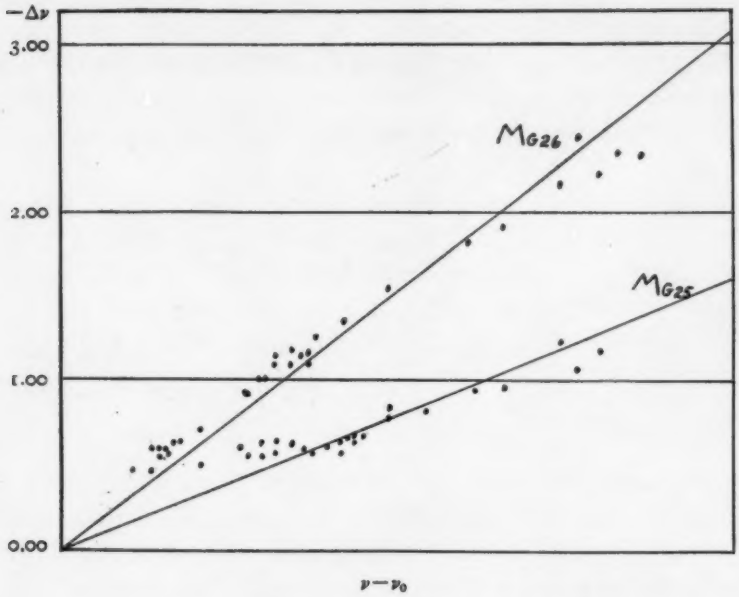


FIG. 2.—Isotope displacements, λ 5211 band

panions which in many cases are measurable lines, but which often are too close to other strong lines to be clearly resolved. The intensity ratios fit the abundance data as closely as can be judged by mere inspection. For several of the series, the displacements of the companions have been tabulated, and are shown in Figure 2, plotted against ν_m. The theoretical values are indicated by the solid lines. The clustering of the points about the two lines is well within experimental uncertainty, and the distance of closest approach of the points to the zero axis, about 0.4 cm^{-1}, correctly measures the resolution on the plates used. The faintness of the lines and the general

complexity of the spectrum are quite sufficient to account for the incompleteness and irregularity of the quantitative data.

The assignment of the band λ 5211 as the (n, n) group may now be supported by a further consideration. If this band represented any other group of vibrational transitions, its isotope companions would be expected to show an easily observed vibrational displacement. The conformity of the points in Figure 2 to a line passing through the origin is sufficiently close to exclude the possibility of any such displacement. The use of half-integral vibrational quantum numbers, introduced by Mulliken for the *BO* bands,[1] while not required, is admissible because the displacements so introduced would be too small to be detected.

THE CARRIER OF THE BANDS

All experiments indicate that the source of the magnesium hydride bands is actually a chemical combination of the two elements, although none such is directly known to exist. By analogy with the heavier alkaline earths, *Ca*, *Sr*, and *Ba*, the natural hypothesis is that of the existence, at least metastably, of a molecule MgH_2. The nature of the chemical bond between hydrogen and a metal is very probably of the same type as that existing between the two atoms in the molecule H_2, both depending upon the existence of some stable configuration of two nuclei and two electrons. With this in mind, it seems possible to think of the formation of MgH_2 as proceeding in two steps: first, the union of a singly ionized magnesium atom, which would have a hydrogen-like character, with a neutral atom of hydrogen, giving an ion MgH^+; and, second, the neutralization of this ion with another electron, and then the addition of the second hydrogen atom.

The purpose of such a speculation is to provide some basis for the hypothesis that the carrier of the bands is a dipolar molecule. This hypothesis is desirable for several reasons. First, the general structure of the partial bands and of the band system is that common to spectra of dipoles, for there is only one ν_0 frequency and but one characteristic moment of inertia. Second, the isotope effect is in agreement with a theory which postulates a dipole as carrier, and it

[1] *Op. cit.*, p. 259, 1925.

is furthermore certain that a linear tri-atomic molecule would give no rotational isotope effect, the magnesium atom being at the center of mass, and a non-linear tri-atomic molecule might well be expected to give several different band systems corresponding to moments of inertia about different axes. Third, the hypothesis of a dipole carrier is desirable in that such a molecule would, speaking chemically, have a quality of unsaturation which, as brought out by Mulliken,[1] is favorable to the production of an electronic band spectrum.

In connection with the last point, a suggestion of possible significance lies in the proximity of the ν_0-point of the band system, at λ 5187, to the green triplet of magnesium, the "*b*" group of the solar spectrum, $\lambda\lambda$ 5167, 5172, and 5183. These lines in the arc spectrum of magnesium correspond to the transitions $2p_i - 1S$, where the final states, $2p_i$, represent the first valence electron in Bohr orbits for which $n = 2$. It seems conceivable that in an MgH^+ ion the magnesium might have a tendency to reassume its second valence electron in its 2p orbit, and that the excitation of this electron to the 1S level would provide the electron transition about which the band system is developed. This involves, of course, the none too probable assumption that the energy-levels in the magnesium atom are left practically undisturbed by the combination with hydrogen, or at least that the two levels in question are displaced by nearly equal amounts. The speculation seems of interest, however, in that little is known at present concerning the relations between the electronic frequencies occurring in band systems and those involved in line spectra.

Supposing the carrier to be a dipole of moment of inertia I, one may, in accordance with the well-known relation, set the coefficient of the linear term of $\Delta f(m)$ equal to $h/4\pi^2 I$. The value of I thus computed from relation (D) applies to the non-rotating, non-vibrating molecule in its final electron configuration. This moment of inertia is 4.6×10^{-40} gm./cm^2 and gives the quite plausible value 1.7×10^{-8} cm for the internuclear distance.

RYERSON PHYSICAL LABORATORY
UNIVERSITY OF CHICAGO
August 1925

[1] *Op. cit.*, **26**, 1, 1925.

SOME STUDIES OF THE STARK EFFECT

By CHARLES E. DEPPERMANN

ABSTRACT

The Stark effect with parallel plates.—A method is described for obtaining the Stark effect by using a long, thin cathode, the discharge being constricted between two parallel plates. The advantage derived in greatly increased light-intensity is found to be discounted by difficulties of construction, of sputtering, and especially of proper focusing. The Stark effect obtained was of about the same order as that for a Lo Surdo tube having diameter equal to the distance between the parallel plates.

An improved type of Stark tube using positive rays is described. The cathode is a long, perforated, hollow cylinder, along the axis of which the auxiliary electrode is placed. The canal rays between these two electrodes are subjected to the electric field. This type of tube can be made to yield as great a depth of light as desired, thus giving greater intensity than the ordinary Stark tube, and affording some possibility of obtaining the Stark effect by absorption for hydrogen and helium.

The Stark effect for the zinc triplet 1s—1p was thoroughly studied under fields from 30,000 to 40,000 volts per centimeter, and with dispersions ranging from 10 to 4 A per millimeter. The Stark effect, if any, was found to be negligible. Nagaoka's positive results are apparently due to the pole effect. Preliminary results for the similar 1s—1p triplet of cadmium and the thallium line $1\pi-1\sigma$ (5350.5 A) also show no appreciable Stark effect.

I. INTRODUCTION

With the theory of the Stark effect quite fully developed, interest therein has considerably waned, although many quite important problems still remain unsolved. For instance, Kramers' theoretical article[1] is still only very partially verified by experiment; the Stark effect in metals has not been fully explored, although it can help much in classifying the spectral lines into series;[2] the Stark effect by absorption has only been touched upon by Ladenburg;[3] the Stark effect could be used to determine how long it takes the electron in the hydrogen atom to get into its perturbed orbit, etc.

It was to prepare the way for the solution of some of these problems that the experiments mentioned in this article were undertaken.

II. THE STARK EFFECT BETWEEN PARALLEL PLATES

1. PURPOSES OF EXPERIMENT

The conditions under which the Stark effect can be obtained are quite stringent, and the intensity of the light is generally rather

[1] *Zeitschrift für Physik*, **3**, 199, 1920; Foster, *Physical Review*, **23**, 667, 1924.

[2] Nagaoka and Sugiura, *Japanese Journal of Physics*, **3**, 45, 1924.

[3] *Zeitschrift für Physik*, **28**, 51, 1924.

feeble, often necessitating long exposures. Any device, therefore, strengthening the intensity would be most welcome. In a Lo Surdo tube, the discharge is constrained near the cathode to pass through a cylindrical hole, ranging from 1 to 10 mm in diameter. While the Stark effect, other things being equal, is inversely proportional to the diameter of the tube used,[1] the light-intensity, however, decreases with the bore, thus discounting the advantage of using a narrow tube. An escape seemed possible by making the discharge pass, not through a hole of cylindrical shape, but between two parallel flat plates placed a few millimeters apart. In this way the light might be obtained from an available dark space, not of a few millimeters, but of 15 cm, or more if desired.

The experiment is also of interest theoretically, for, although one dimension (the distance between plates) is small, as in tubes of cylindrical bore, yet the depth is very great in comparison. Some doubt could therefore be raised as to whether conditions for a marked Stark effect were present. Another possible use of the tube might be to obtain thereby the Stark effect by absorption, by sending the light from a source of continuous spectrum through the long depth of dark space.

2. APPARATUS

After many models had been made and discarded, mainly because of lack of durability, the following type proved fairly successful.

LMNO (Fig. 1*a*) is a side view of a fused silica plate, 11 cm long, 7 cm wide, and 4 mm thick. Soda glass and even pyrex plates cannot be used, for they soon crack with the intense heat of the discharge at the cathode. Two such plates are used, with the cathode *D* and the anode *E* in between.

These electrodes are aluminium strips 10 cm long, 5 mm wide, 1.5 mm thick. Since the discharge will seek to spill in all directions rather than be constricted into the narrow space between the plates, the cathode must be hedged in very carefully. This is accomplished as follows: (1) the glass strip *A* is placed between the cathode and the edge of the plates (Fig. 1*a*); (2) thin glass plates *F* and *G* are fas-

[1] Takamine and Yoshida, *Memoirs of the College of Science* (Kyoto Imperial University), **2**, No. 6, 1917; Brose, *Annalen der Physik*, **58**, 731, 1919.

tened to the ends of the silica plates by a small amount of glass glue and by the brass clamps *X* and *Y*, which latter also serve to hold the silica plates tightly together; (3) to avoid sputtering upon *F* and *G*, the cathode is separated therefrom by the small glass pieces *B* and *C*; *A*, *B*, and *C* are held in place by the pressure of the silica plates and a little glass glue; (4) to prevent the discharge creeping from the

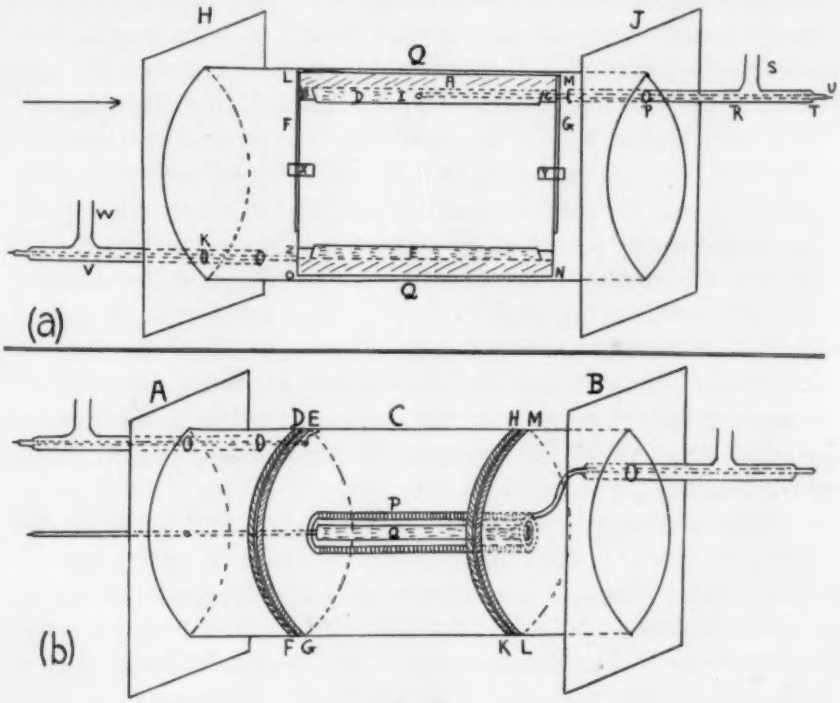


FIG. 1

a) Discharge tube for Stark effect between parallel plates
b) Stark tube using perforated cylinder

cathode to any aluminium sputtered upon the silica plates, the edge of the cathode *D*, facing the anode, should be milled a little, so that it does not touch the plates.

The anode need not be so well protected as the cathode.

This whole unit is now placed in a pyrex cylinder (*Q*, Fig. 1*a*) about 16 cm long, 7.3 cm in diameter, with walls about 2–3 mm thick, and held tightly therein by glass glue. etc. The ends of *Q* are

backed by the pyrex plates H and J, which are 3 mm thick, 10 cm square. Soda glass was found unsuitable because it cracked too easily.

Considerable care must be exercised in insulating properly the leads coming from the electrodes, to prevent leakage of the discharge. I (Fig. 1*a*) represents a hole bored into one of the silica plates, out of which runs the cathode lead incased in glass tubing of small bore, the end of which is well ground into the hole I. The tubing passes through hole P, ground in plate J, and then through glass tubing of larger bore R. Points U, P, and T are sealed vacuum-tight with sealing wax. The apparatus is exhausted through the side tube S in R. The anode lead is similarly safeguarded, the side tube W being used for the inlet of the desired gases. The whole instrument should be cooled by a water-bath or by a fan to prevent the wax from melting. The discharge is viewed end-on in the direction of the arrow (Fig. 1*a*).

Well-baked asbestos board or lavite could be used instead of the fused silica. The essential point is to have the cathode and the dark space well hedged in, so that the discharge, within 5 mm to 1 cm or so from the cathode, can pass only through the narrow constricted space between the two parallel plates.

The spectroscope and the apparatus for obtaining direct current of high voltage are substantially those described later (see p. 41).

If a grating be used, in order to get sufficient resolving-power, the narrow emergent beam of light should be spread out by a cylindrical lens so as to fill the grating, while leaving the beam intense and unmagnified in the other direction. If prisms are used, such lens is, of course, unnecessary.

3. RESULTS

a) *The Stark effect obtained.*—With a single-prism spectroscope no Stark effect could be noticed until the plates were set only 1 or 2 mm apart and a comparatively heavy current of 20–50 mil. amp. or more was used. With a grating of 30,000 lines and the set-up to be described later (Fig. 3*b*, *c*), it was found, by visual observation, that, with the plates about 1.5 mm apart, and using a current of 20 mil. amp. and a pressure of about 0.3 mm, the Stark effect was approxi-

mately equal to that obtained with a cylindrical tube of 3-mm bore, using a few milliamperes current. Though the exact relation of current to Stark effect, other things being equal, does not seem to have been accurately determined as yet,[1] even for the ordinary Lo Surdo tube, it appears quite safe to say that the foregoing experiment shows that the Stark effect between parallel plates is of the same order of magnitude as in cylindrical tubes whose diameter equals the distance between the parallel plates, but that the increased area of cathode used demands a corresponding increase in the current. However, the difficulties described below prevented closer study and even the taking of satisfactory photographs.

b) *Difficulties.*—Though quite splendid intensity of light was procured, grave drawbacks were encountered which seem to more than offset this advantage. In the first place, the proper elimination of sputtering upon the windows *F* and *G* (Fig. 1*a*) is very difficult, even using the glass strips *B* and *C*, and yet these windows seem necessary to keep the discharge from spilling. A molybdenum cathode might prove more satisfactory than the aluminium used.

In the second place, proper focusing of the discharge upon the slit of the spectroscope is well nigh impossible. In the Lo Surdo type of tube, the electric field, and therefore the Stark effect, varies for different distances from the cathode; hence, in order not to get a confused medley upon the photographic plate, accurate focusing is absolutely essential. In the case of the flat discharge tube, with its long depth of dark space, no lens system can accurately focus the whole extent. The lens might possibly be placed far from the tube, but then the image would be extremely small. Again, if, by means of a system of parallel slits, the light from only one level in the dark space be singled out for investigation (preferably the level of maximum Stark effect, about 0.5 mm from the cathode), the intensity of the light is greatly diminished and the main advantage of the flat discharge tube lost.

It may be concluded, therefore, that the flat discharge tube is beset with too many difficulties to make the gain in intensity of light profitable.

If, however, it be desired for theoretical reasons to study this

[1] Takamine and Yoshida, *loc. cit.;* Brose, *loc. cit.*

type of discharge further, the following model is suggested, as being most easy to take apart and vary. Two lavite plates, about 5 cm long, 2 cm wide, and 1 cm or so thick, should be procured, and a molybdenum cathode confined between them in essentially the same manner as already shown in Figure 1. This unit is then set upon the top of the silica tube *D* (Fig. 2) used in Anderson's bell-jar method (see p. 40). The anode need be only a long aluminium wire set parallel to the cathode and 3 or 4 inches above it. It is readily seen that this type admits of quick changes, and is well adapted for experimental purposes.

III. A NEW TYPE OF STARK TUBE

1. PURPOSE OF EXPERIMENT

It is in the Stark method of imposing an electric field upon the positive rays that feeble light-intensity is most keenly felt, the more so since this method yields such beautiful results. The following variation from the ordinary Stark tube is offered, not only as affording more depth of positive rays, but also as giving some possibility for obtaining the Stark effect by absorption.

2. DESCRIPTION OF TUBE

The inside of a pyrex cylinder (Fig. 1*b*), 15 cm long, 7.5 cm in diameter, with walls 2–3 mm thick, is lined, for the middle 12 cm of its length, with sheet aluminium just thick enough (about 0.15 mm) to retain its shape quite well around the inside of the tube. The cathode *P* is a hollow aluminium cylinder 12 cm long, 1 cm inside and 1.3 cm outside diameter. It is perforated with small holes 1 mm in diameter and about 3 mm apart, except for a space of 1 cm or so from the ends of the cylinder (to prevent sputtering upon the plates *EG* and *HK* [Fig. 1*b*]).

Accurately centered along the axis of the cathode is the cylinder *Q*, 12 cm long, 0.6 cm in diameter. This auxiliary anode affords a space 2 mm wide between itself and the cathode, in which the canal rays are subjected to a uniform electric field by connecting the ends of the two anode leads together.

To avoid spontaneous discharges between the auxiliary anode and the cathode, thus destroying the electric field, certain precau-

tions must be carefully taken. The perforated cathode has its ends set accurately into holes drilled through the centers of two pyrex plates, which fit just a little loosely into the outer pyrex cylinder, so as to hold the anode sheet between them and the cylinder. The cathode lead is led out through small glass tubing as shown in Figure 1*b*. The auxiliary electrode must not touch the above-mentioned glass plates at all. A pin in one end of this electrode fits into a small hole 1 mm in diameter, drilled halfway into a circular plate (*LM*, Fig. 1*b*) similar to *HK* and concentric with it, but held a fraction of a millimeter therefrom by some glass glue between the rims of the glass plates. Hence no spontaneous discharge can seep from the cathode to the auxiliary anode, for they do not touch directly at all. On the other side is a plate similar to *EG*, but with a central hole just large enough to let out the auxiliary anode lead in a small glass tubing. The auxiliary anode must be accurately centered inside the cathode; otherwise, no uniform field can be obtained. The exhaust tube and tube for inlet of gases are arranged in general as in the flat discharge tube. When run with heavy current, water-cooling is essential; for small current, a fan will suffice.

3. RESULTS

In spite of the long depth of canal rays, the light was found not to be very strong at best, though it still is an improvement over the other types using canal rays. Its use cannot therefore be urged for reasons of intensity of light obtained. However, it should be possible to adapt this type to the study of the Stark effect by absorption for hydrogen or for helium, since light from a source yielding a continuous spectrum could be sent through the space between auxiliary anode and cathode. The advantage of this type of tube over others is that the column of absorbing gas can be made as long as we please by simply lengthening the cathode and auxiliary anode. A long column would certainly be needed for hydrogen, since Hulburt's condensed discharge method[1] would cause spontaneous discharge between the cathode and auxiliary electrode. Furthermore, the positive rays are most intense at rather low pressures, i.e., about 0.1 millimeter.

[1] *Physical Review* (2), **23**, 593, 1924.

IV. THE STARK EFFECT IN METALS OF LOW MELTING-POINTS

1. INTRODUCTION

In 1917 J. A. Anderson[1] devised a type of apparatus (the so-called bell-jar method) suitable for study of the Stark effect in metals of comparatively high melting-points. He himself studied chromium carefully, and was followed by Takamine[2] with the Stark effect in silver, gold, copper, cobalt, iron, manganese, magnesium, molybdenum, nickel, and tungsten. The metals of quite low melting-points had not yet been tried by this method. An attempt was therefore made to obtain the Stark effect for zinc, cadmium, antimony, thallium, etc. Considerable difficulty was encountered, due to prolific sputtering, and only zinc has been at all carefully studied, and that solely for the zinc triplet $1s-1p$ in Fowler's notation (4680.2, 4722.2, 4810.5 A).

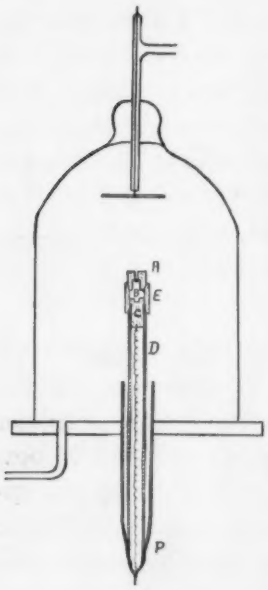


FIG. 2

A, E, D—Fused silica
B—Removable metal cap for cathode.
C—Permanent iron cathode base.

2. DESCRIPTION OF APPARATUS

The type of apparatus was essentially that of Anderson, but the following rather slight changes should be noted.

a) It was found difficult to restrain the discharge from seeping through the ground joint *P* (Fig. 2), thus causing intense local heating and breakage of glass. To overcome this, a rough joint only was made at this point, then a little glass wool tamped between the silica tube and the outer glass tubing; upon this, dry alundum cement was tamped down, to within 5 mm of the top of the glass tube, and then finally a cap of wet alundum cement put on top to keep things firm. This prevented seepage of current almost entirely.

b) To obtain larger Stark effects if possible, tubes were used of

[1] *Astrophysical Journal*, **46**, 104, 1917. [2] *Ibid.*, **50**, 1, 1919.

smaller bore than those of Anderson. Figure 2 shows how the various parts of the cathode and its protecting caps were fitted together. Table I gives the bore of the constricted region close to the cathode used for the various photographs. It was found that the drilling of holes and making of ground joints could be done much more expeditiously with crushed steel than with emery.

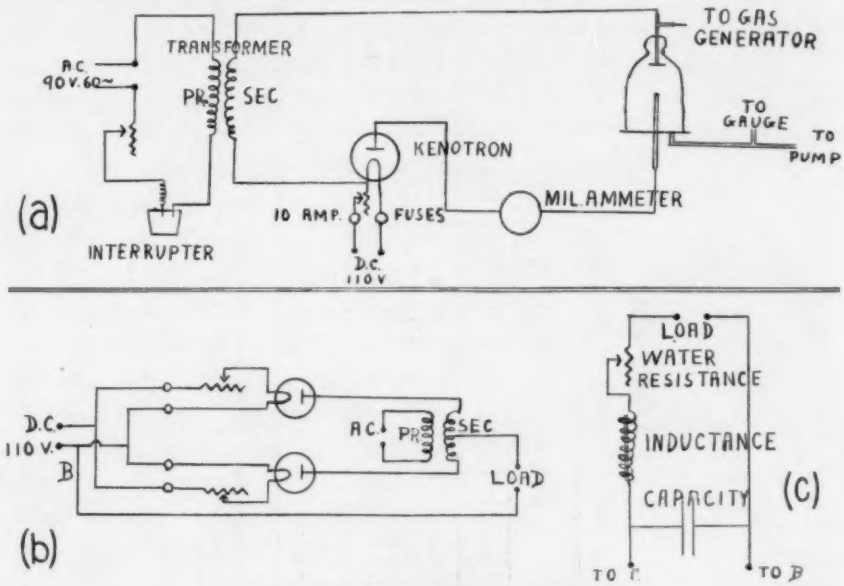


FIG. 3.—Various arrangements used

c) The current was obtained from an A.C. generator (double-current type, 60 cycles) giving 90 volts, then raised to about 5000 volts by a transformer, and rectified by either 1 or 2 kenotrons (100 watts, 12-volt filament current, and capable of rectifying 0.1 amp., 20,000 volts). Figure 3 gives details of the various arrangements used. The capacity employed in the last photograph for zinc consisted of seven large condensers (each of 0.01 microfarad, 10,000 volts).

3. RESULTS FOR ZINC

Nagaoka gives a photograph of the zinc spectrum taken at the anode of his stabilized arc in air[1] (Fig. 4), in which the zinc triplet

[1] Nagaoka and Sugiura, *loc. cit.*

TABLE I

Zinc Triplet 1s—1p

No. of Photograph	4810.5 A		4722.2 A		4680.2 A		Max. Field Volts per Centimeter	Approx. Pressure in Millimeters of *Hg*	Current* (in Milliamperes)	Exposure in Hours	Resid. Gas	Bore near Cathode in Millimeters
	p-comp.	*s*-comp.	*p*-comp.	*s*-comp.	*p*-comp.	*s*-comp.						
1.....	0.53 A	0.72 A	0.48 A	0.60 A	0.40 A	0.40 A	32,000	0.8	5 (int.)	1¼	*H*	5.5
2.....	.60	.72	.49	.59	.38	.38	30,000	3.4	10 (int.)	½	Air	5.5
3.....	0.75	.86	.61	.72	0.57	0.57	40,000	1.5	2–3 (cont.)	2	*H*	3
4.....	1.04	.93	.83	.73	Too faint to measure	Too faint to measure	35,000	3.0	1 (cont.)	2	Air	3
5.....	0.6	.6	.....	.....			35,000	0.7	A few	2	*H*	3
6.....	0.36±	0.36±	0.36±	0.36±	.....	.....	40,000	1.7	2 (cont.)	2	*H*	2.5
7†.....	.....	.....	.....	.....	.....	.....	40,000+	.....	5 (cont.)	2	*H*	2.5

* Direct current was used for all except photograph No. 5.

† Wide slit used to get intensity enough for possible faint side components. No such components found.

For photographs Nos. 1–5, dispersion = 10 A per mm, 1 kenotron used; for No. 6, dispersion = 4 A per mm, 2 kenotrons, but no inductance or capacity; for No. 7, dispersion = 4 A per mm, 2 kenotrons, also inductance and capacity.

1s—1p seems to show a pronounced Stark effect, surprisingly great, in fact, for the first lines of a principal series. Hence, it was thought advisable to try for the Stark effect upon this zinc triplet under the more ideal conditions of the discharge *in vacuo*, where the disturbing factors, which contribute to make up what is known as the pole effect and which tend to mask a pure Stark effect arising only from an external field, are much less in evidence.

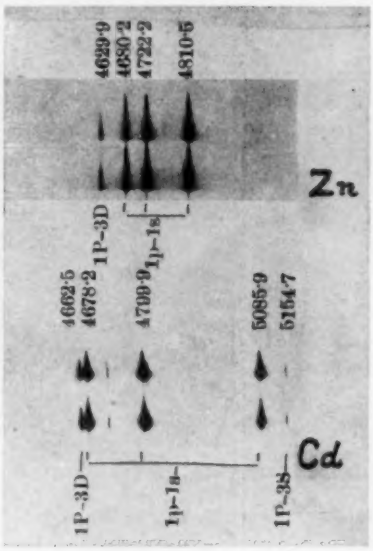


FIG. 4.—Zinc and cadmium triplets at anode of stabilized arc in air according to Nagaoka and Sugiura.

Since the melting-point and boiling-point of zinc are quite low (419° and 930° C., respectively, in air) it was deemed advisable not to use pure zinc as the cathode; brass (*Cu* and *Zn*), with melting-point of about 960° C., was therefore substituted. A current of over 10–20 mil. amp. was first tried, but this proved to be too great. The prolific sputtering in very short order lined the sides of the silica cap and the slit itself with metal, from which the discharge commenced, thus destroying any Stark effect. Hence a current of a few milliamperes or less was used; in some cases, also, the current was interrupted every second or so by means of a vibrating spring dipping into a cup of mercury (Fig. 3*a*). Even with these precautions, the time of discharge had to be limited to two hours, for by this time a very considerable hole had been gouged in the top of the cathode. As a consequence of this time limit, only the triplet 1s—1p usually appeared on the plate.

Table I gives the widths of this triplet as found upon the various plates, together with the main conditions under which the photographs were taken.

A careful consideration of the foregoing data forces one to the inevitable conclusion that the Stark effect upon the first triplet of

the principal series of zinc is negligible in a discharge tube even up to a field of about 40,000 volts per centimeter. The width of the two stronger lines of the zinc triplet is less in the photograph taken with a dispersion of 4 A per millimeter than in that taken with 10 A. Whatever slight broadening is shown on any of the plates can easily be explained as due to the intensity of the light in the dark space.

TABLE II

Element	Lines	Stark Effect	Authority
Mg.....	$1p-1s$, sharp series triplet	No data; diffuse series $1p-2d$ with 30,000 volts/cm gives but 0.1 A	Takamine*
Mn.....	$1p-2s$, sharp series	Not affected	Takamine*
Cu.....	$1\sigma-1\pi$, principal series doublet	Big spread in photograph; no data given	Nagaoka†
Ca.....	H and K lines ($1\sigma-1\pi$) of ionized *Ca*	2.3 and 2.6 A, respectively, but not found by others	Howell‡
	Other lines	Negligible	Takamine,§ Stark and Kirschbaum||
Hg.....	$2p_1-ms$ (4 lines of sharp series)	No effect	Hansen, Takamine, and Werner¶
Al.....	$1\pi-2\delta$, $1\pi-3\delta$	Only 0.3 and 0.2 A on these diffuse lines	Wendt and Wetsel**
		Negative results	Howell‡
Tl.....	$1\pi-1\sigma$ (sharp)	No effect	This article
Cd.....	$1s-1p$ (principal)	No effect	This article
		Large spread in photograph; no data given	Nagaoka†

* *Astrophysical Journal*, **50**, 1, 1919.
† *Japanese Journal of Physics*, **3**, 45, 1924.
‡ *Astrophysical Journal*, **44**, 87, 1916.
§ *Memoirs of the College of Science* (Kyoto Imperial University), **3**, No. 6, 1918.
|| *Annalen der Physik*, **43**, 1017, 1914.
¶ *Danske Vidensk. Selskabs., Math.-Phys. Meddelelser*, **5**, No. 3, 1923.
** *Annalen der Physik*, **50**, 419, 1916.

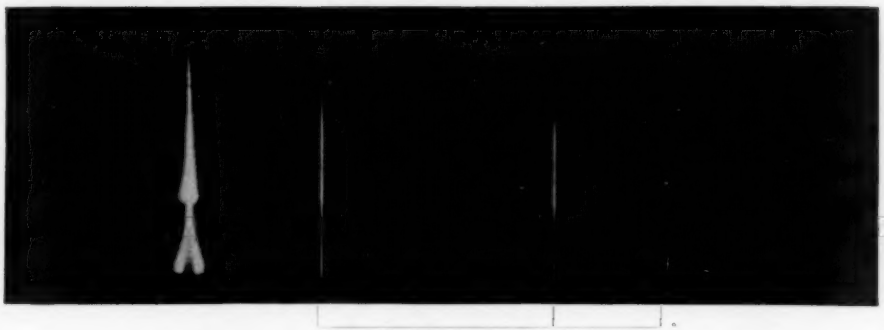
Plate II*c*, taken with a rather wide slit, to obtain strong lines, precludes the possibility of fainter side components to the main lines.

These results seem quite in accord with those of other authors upon lines of the principal or sharp series of other elements, summarized in Table II.

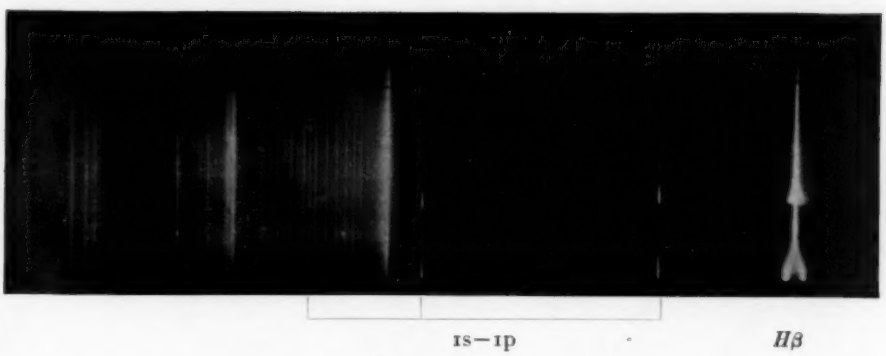
Another confirmation is found in the article by Miss Howell,[1] who writes: "*Fe, Ni, Mg, Al, Zn*, and *Sr* show *no* electric effect." The spectroscope used, however, had a small dispersion (12–18 A

[1] *Astrophysical Journal*, **44**, 87, 1916.

PLATE I



No. 3 of Table I



No. 4 of Table I

STARK EFFECT FOR ZINC TRIPLET

UNIV. OF MICH.

PLATE II

Alternating current

No. 5 of Table I

No. 6 of Table I

No. 7 of Table I

STARK EFFECT FOR ZINC TRIPLET

UNIV. OF MICH.

per mm), and the field obtained appears to have been only about 20,000 volts per centimeter, whereas my final results were taken with a dispersion of about 4 A per millimeter, and a field of about 40,000 volts per centimeter.

How, then, can we account for the marked Stark effect apparently obtained by Nagaoka for the zinc and cadmium triplets $1s-1p$? Two explanations seem at present possible: either Nagaoka's result was due to the mixture of causes giving rise to what is known as the pole effect, and hence not due to a purely external electric field but rather to extraordinarily large electric fields between the zinc atoms themselves, or else, as sometimes happens, the reproductions may be poor. A definite answer must await the original data of Nagaoka himself, who has been written to therefor, since no data accompany the photograph.

While trying to vary the conditions as much as possible, a few photographs were taken in an endeavor to see what Stark effect could be obtained with alternating current instead of direct (see Plate II*a*).

Some interesting points were noted:

a) Though the exposure time was two hours, the picture is fainter than that obtained with direct current; this is especially true of the metallic lines. It was also noted that the heat generated at the cathode was less. Evidently alternating current is not propitious for bringing out metallic spectra, though the Stark effect, at least for the hydrogen lines, can be obtained.

b) The curving-in of the components of the hydrogen lines, so noticeable in direct-current pictures and so well described in the article by Brose,[1] does not seem so much in evidence when alternating current is used. However, more careful investigation would be necessary to determine whether this is a general phenomenon or an accident. If it be a bona fide result, it might possibly be explained on the theory that the rapid change of current prohibits the formation of the double layer mentioned by Brose and by Nagaoka.[2]

An interesting idea here suggests itself. Could not the alternations of current be made extremely rapid and a frequency finally

[1] *Loc. cit.*

[2] Brose, *ibid.*; Nagaoka and Sugiura, *loc. cit.*

reached which would not permit the atom to get its electron into the disturbed orbit and so destroy the Stark effect? The idea seems capable of experimental determination.

It may be mentioned, in passing, that only in one set of circumstances, i.e., with heavy direct current with air as the residual gas, was a condition at the anode found at all similar to that obtained by Nagaoka with his stabilized arc in air.[1] Under these conditions the discharge does not cover all the circular anode plate, but comes only from one or more very bright, intense points (see Anderson),[2] spreading out radially therefrom. No attempt was made to photograph their spectrum, since they rarely remained long in any single position. The matter should be further investigated, as the bright spots seem somewhat like those described by Nagaoka.

4. PRELIMINARY RESULTS FOR CADMIUM, ANTIMONY, AND THALLIUM

a) *Antimony.*—Though quite a few pictures with an antimony cathode were attempted, yet it is surprising that there are very few, if any, indications of antimony lines upon the plates. The explanation may be twofold; antimony forms a gaseous compound with hydrogen called stibine (SbH_3); again, the vapor of antimony is generally polyatomic.

b) *Thallium.*—Thallium used as a cathode yields a brilliant green in the dark space, and the line 5350.5 A comes out strongly upon the plate, but very sharp and with no trace of a Stark effect visible. The line was intense enough to be subjected to the scrutiny of an echelon, but even here an examination (not very exacting) seemed to show no perceptible broadening.

c) *Cadmium.*—This is a prolific sputterer even at currents as low as 1 mil. amp. As a consequence, though three pictures showed the cadmium triplet $1S-1P$ and the line $1P-3D$ strongly (with no visible Stark effect), still they also showed that, due to sputtering, the discharge had come from the sides of the silica cap and from the slit, as well as from the true cathode level.

d) *Lead and tin.*—Two photographs with a solder cathode gave no results, as the discharge almost immediately came from the sides of the silica cap.

[1] *Loc. cit.* [2] *Loc. cit.*

These are rather hasty attempts, but they seem, however, to show definitely that great care must be taken with metals of low melting-points, if their Stark effect is to be obtained. Alloys of these metals should be sought with others of higher melting- and boiling-points; the use of brass instead of pure zinc is a case in point. If these alloys cannot be obtained, then the brilliancy of the spectrum should be enhanced by using a halogen compound of the metal as an anode together with the pure metal as cathode, as was done by Kimura and Takamura.[1] The more intense light obtained would permit the use of a smaller current, and thus give a reduction of sputtering.

The author wishes to express his thanks and deep appreciation to Dr. Pfund for his continued interest and advice during this work, and also to Dr. Ames and Dr. Anderson for their timely suggestions.

JOHNS HOPKINS UNIVERSITY
June 9, 1925

[1] *Japanese Journal of Physics*, 3, 45, 1924.

THE ATMOSPHERE OF MARS

By DONALD H. MENZEL

ABSTRACT

The appearance of a planet depends on the selective absorption and scattering of its atmosphere. Using the known values of the visual and photographic albedos, it is possible to evaluate the maximum quantity of atmosphere which Mars possesses. *The amount per unit surface* is not greater than one-fifth that above the earth. Under the lesser gravity, *the corresponding pressure* will be less than 5 cm of mercury—equivalent to the pressure in the terrestrial atmosphere at a height of 18 km.

The foregoing amount of atmosphere is insufficient to account for the appearance of *Wright's photographs* (*Publications of the Astronomical Society of the Pacific*, **36**, 234, 1924), which showed no detail in the ultra-violet, but strong contrast in the red. This is probably a characteristic of the material which composes the planet.

The Martian polar caps are not an atmospheric but a surface phenomenon. Their relative prominence in the two pictures is due mainly to the difference in visual and photographic albedos.

The greenish color of Uranus and Neptune are due to the absorption, by their atmospheres, of the red end of their spectra.

The recent opposition of Mars has shown the importance of photography in interpreting physical conditions on the planets. Photographs taken in different colors reveal a wide range of contrast, presumably due to the character of the surface and atmosphere.[1]

Up to this time the influence of the atmosphere on the appearance of the planet has been considered only in a qualitative way. Believing that a quantitative discussion of the matter would be of value, I have undertaken to determine the contribution of a planet's atmosphere to the total light from the planet.

THEORY

Let the total energy, between a small range of wave-lengths, incident on the planet's atmosphere be denoted by I. Of this a fraction x is scattered back to space and a fraction x' is absorbed. Thus Ix is reflected from the atmosphere and $I(1-x-x')$ will reach the planet. Similarly, if J denotes the amount of energy which goes out from the planet, the amount sent back by the atmosphere will be Jx and that which gets out to space $J(1-x-x')$. Therefore the total

[1] Wright, *Publications of the Astronomical Society of the Pacific*, **36**, 239, 1924; Hubble, *ibid.*, Plate XXIII.

light received by the planet is $I(1-x-x')+Jx$. Let y represent the albedo of the surface for the light in question, then

$$J=yJx+yI(1-x-x')\,,$$

or

$$J=\frac{y}{1-xy}\,(1-x-x')I\,.$$

That part which escapes from the planet to space

$$=J(1-x-x')=\frac{y}{1-xy}\,(1-x-x')^2I\,.$$

When we add to this the contribution by the atmosphere, the total light reflected is

$$fI=Ix+\frac{y(1-x-x')^2}{1-xy}\,I$$

$$f=x+\frac{y(1-x-x')^2}{1-xy}\,,$$

where f, the fraction of the incident energy eventually returned to space, is evidently the planet's albedo for light of the particular color under consideration.

Expanding and collecting terms,

$$f=\frac{x+y-2xy}{1-xy}-x'y\,\frac{2-2x-x'}{1-xy}\,. \qquad (1)$$

All the quantities f, x, x', and y are less than one and greater than zero, and are functions of the wave-length of the light under consideration. If f is constant for all wave-lengths, the character of the sunlight would be unaltered by reflection. If f increases with wave-length, the planet will appear red; if it decreases, blue. Equation (1) may be written in the form

$$f=y+\frac{x(1-y)}{1-xy}-\frac{x'y(2-2x-x')}{1-xy}\,. \qquad (2)$$

For the simplest case, assume simple Rayleigh scattering, x inversely proportional to the fourth power of the wave-length, and y constant. If there is no absorption present, equation (2) shows that f will increase with x. This is a formal proof of the somewhat obvious fact

that a "gray"[1] planet with a selectively scattering atmosphere will always appear blue. For the planet to appear blue with absorption only ($x=0$) requires increased absorption for the red, and vice versa.

APPLICATION TO MARS

In applying the foregoing to Mars, the assumption that $x'=0$, both for that planet and the earth, is probably near the truth.

Let $y=af$, where a is a constant which represents the ratio of the albedo of the surface to the total albedo of the planet.

From equation (1)

$$x=\frac{f-y}{1-(2-f)y}. \tag{3}$$

Combining,

$$x=\frac{f(1-a)}{1-2af+af^2}. \tag{4}$$

In the following, the subscripts p and v are used to denote the specific values of x, f, and a for photographic and visual light, respectively. The effective wave-length for the former is approximately 4400 A, and for the latter 5600 A. For Rayleigh scattering, x is inversely proportional to the fourth power of the wave-length, therefore

$$x_p=3x_v. \tag{5}$$

Using this relation in connection with equation (4), the following relation is derived:

$$\frac{f_p(1-a_p)}{1-2a_pf_p+a_pf_p^2}=3\,\frac{f_v(1-a_v)}{1-2a_vf_v+a_vf_v^2}. \tag{6}$$

For Mars, the albedos f_p and f_v are equal to 0.09 and 0.15, respectively.[2] Substituting the numerical values, equation (6) reduces to

$$\frac{0.09(1-a_p)}{1-0.17a_p}=\frac{0.45(1-a_v)}{1-0.28a_v}.$$

Solving,

$$a_v=0.85+\frac{0.63a_p}{4.72-0.57a_p},$$

or $a_v>0.85$.

[1] y=constant.

[2] Russell, *Astrophysical Journal*, 43, 173, 1916.

From equation (3), $x_v < 0.03$ and, using equation (5), $x_p < 0.09$ Table I gives various possible values of x and y computed from equations (3) and (5). It is necessary to evaluate x, the total scattering over a hemisphere in terms of the fraction scattered perpendicularly from the zenith (unit air-mass).

TABLE I

x_v	0.03	0.02	0.01
x_p	.09	.06	.03
y_v	.12	.13	.14
y_p	0.00	0.03	0.06

Figure 1 represents the appearance of the planet, the inner and outer circles marking the boundaries of the surface and atmosphere, respectively. Let r be the radius of the planet. The solar rays are incident from above, the shading denoting the night hemisphere. The distance CB represents unit air-mass, which scatters the fraction Q of the incident energy. The amount of scattering over any other distance, DA, will be approximately proportional to the length of the path. The angle of incidence of the solar rays upon the planet will be ϕ. Thus the reflecting power of any point of the atmospheric surface is $Q/\cos\phi$, i.e., constant on concentric circles perpendicular to the direction of the incoming solar energy. Consider the planet's surface. A circular element of area of constant albedo about OC, as an axis, is $2\pi r^2 \sin\phi d\phi$. Since the surface is inclined to the direction of the beam, the intensity of the incident radiation per unit surface is $l \cos\phi$ where l is the intensity perpendicularly incident on unit area. Therefore, the total amount of energy falling on the strip is

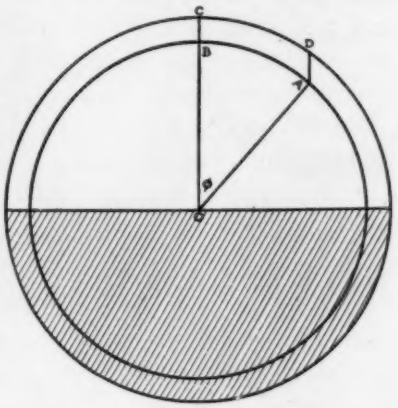


FIG. 1

$$2\pi l r^2 \sin\phi \cos\phi d\phi .$$

Integrating between o and $\pi/2$, the total energy falling on the planet is πlr^2.

The light reflected from the strip will be $2\pi lr^2 Q \sin \phi d\phi$, which integrated gives, from the entire planet, $2\pi lr^2 Q$. Taking the ratio of the reflected to incident energy, the effective albedo of the atmosphere $x = 2Q$.

This is the extreme case. Actually it will be less than $2Q$. Schuster[1] has shown that

$$k = \frac{1}{1+\frac{st}{2}}, \tag{7}$$

where k is the fraction of light from the incident beam which is transmitted through a scattering atmosphere of thickness t. The scattering coefficient is s. Since half the scattered light goes upward and half downward, for air-mass 1,

$$\frac{st}{2} = Q,$$

and for any other air-mass

$$\frac{st}{2} = \frac{Q}{\cos \phi}.$$

The fraction reflected back to space equals

$$1-k = \frac{\frac{Q}{\cos \phi}}{1+\frac{Q}{\cos \phi}},$$

which represents the effective albedo at any point.

As before, the light falling on the strip is

$$2\pi lr^2 \sin \phi \cos \phi d\phi.$$

That reflected is

$$\frac{2\pi lr^2 Q \sin \phi}{1+\frac{Q}{\cos \phi}} d\phi,$$

[1] *Astrophysical Journal*, **21**, 6, 1905.

or the total reflected from the planet's entire hemisphere is

$$2\pi lr^2 Q \int_0^{\frac{\pi}{2}} \frac{\sin\phi\cos\phi}{Q+\cos\phi}\, d\phi = 2\pi lr^2 Q[1+Q\log Q - Q\log(1+Q)] .$$

The total incident light is πlr^2.

The ratio of these two quantities gives the atmospheric contribution to the total albedo

$$x = 2Q[1+Q\log Q - Q\log(1+Q)] . \qquad (8)$$

Table II gives x for different values of Q.

TABLE II

$Q=$	0.2	0.1	0.05	0.025	0.02	0.01
$x=$	0.26	0.15	0.085	0.045	0.038	0.019

THE ATMOSPHERE OF MARS

From these and other considerations an upper limit to the quantity of atmosphere on Mars is estimated. Above Mount Wilson, for unit air-mass, 0.20 of the energy at λ 4400 is scattered out of the direct beam of the sun.[1] Of this, 0.10 goes upward and 0.10 downward. Therefore $Q_e = 0.10$, the subscript referring to the earth. Referring to Table I, it is shown that $y = 0$ if $x_p = 0.09$. This would require a surface entirely black for short waves, a condition not found in nature. Wilsing and Scheiner[2] have measured many rocks spectrophotometrically. While they have not observed the reddest rocks known, in no case is the photographic reflectivity less than half the visual. If, as an extreme, it is assumed that the average rocks on Mars are as red as those found in the region of the Arizona "painted desert," it is possible that the ratio of visual to photographic albedo might approach three, that is,

$$y_v = 3y_p . \qquad (9)$$

Table I shows that this will occur only when $x_p = 0.045$. From Table II, Q, for this value of x, equals 0.025, just one-fourth of Q_e. If the scattering were directly proportional to the masses of the atmos-

[1] *Smithsonian Physical Tables* (7th rev. ed.), p. 418.

[2] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **20**, No. 61, 1909; *ibid.*, **24**, No. 77, 1921.

pheres, the quantity of the atmosphere per unit area above the surface of Mars would be one-fourth that above Mount Wilson, or one-fifth that above sea-level, since the pressure at the higher altitude is four-fifths the sea-level pressure.

More accurately, applying the Schuster approximation, equation (7), the fraction scattered for the earth by unit air-mass ($t=1$) is

$$Q_e=1-k=\frac{\frac{s}{2}}{1+\frac{s}{2}},$$

or

$$\frac{s}{2}=\frac{Q_e}{1-Q_e}=0.11\ .$$

The same equation may be applied to Mars, to determine t, the thickness of its atmosphere per unit area in terms of the earth's as unity, assuming the scattering coefficient, $s/2$, to be approximately the same for both.

$$Q=\frac{\frac{s}{2}t}{1+\frac{s}{2}t},$$

therefore,

$$t=\frac{Q}{\frac{s}{2}(1-Q)}=0.23\ .$$

Multiplying by $\frac{4}{5}$ as before to reduce to sea-level, it is found that the quantity of atmosphere per unit area on Mars cannot be greater than 0.18 that of the earth. The Schuster approximation thus introduces no great improvement in determining the atmospheric thickness on Mars.

The pressure at the surface of the planet varies as the product of the quantity of material and the surface gravity. For Mars the later factor equals 0.38. The total pressure, as measured by an aneroid barometer, cannot be greater than 5–6 cm of mercury. This is equivalent to the pressure in the terrestrial atmosphere at a height of 17–18 km (11 mi.) above the surface of the earth.

This is a maximum value for the atmospheric pressure on Mars. It may be less, according to the degree of redness assumed for the planet's surface. If $x=0.02$, $Q=0.01$, giving for $\frac{4}{5}t$ the value 0.07, corresponding to a pressure of only 2 cm of mercury.

Without additional assumptions, the minimum quantity of atmosphere cannot be evaluated. Even the maximum quantity here deduced is insufficient to account entirely for the variations in Wright's observations. His photographs show the blue almost devoid of detail, which effect he considers atmospheric.[1]

The foregoing discussion shows that the photographic brightness of the image of the planet's surface must at least be equal to that of the atmosphere (i.e., $x_p \geqq y_p$). The observed effect is quite in accord with theory. The spectrophotometric measures of Wilsing and Scheiner[2] show that many rocks, while having widely different values of the albedos in the red, show little contrast in the photographic region. In support of this argument it should be stated that some of the photographs in the blue do show a faint indication of surface markings. The impression, however, which one receives from the pictures is that part of their appearance is an atmospheric phenomenon. The red photographs show a fading toward the edge which apparently cannot be explained on any other basis.

Wright also calls attention to the fact that the blue images are larger than the red, and attributes this radial excess to the overlying atmosphere.[3] The maximum quantity of atmosphere just derived is insufficient to give an effect of the magnitude Wright observes. The edge effect just mentioned suggests that the inconsistency may be due as much to a radial deficiency in the red images as an excess in the blue. Further investigation is desired, and it should not be difficult to prove whether the result is entirely a planetary condition or whether it should be attributed in part to irradiation or other photographic causes.

Lowell's reduction of the observations of Douglass[4] is additional evidence that such an effect should be small. He finds that a twilight arc of 10° is present on Mars, i.e., the sun's rays continue to reach

[1] *Op. cit.*, p. 246.

[2] *Loc. cit.*

[3] *Op. cit.*, p. 250.

[4] *Astrophysical Journal*, **2**, 136, 1895.

the planet's surface until the sun has sunk 10° below the Martian horizon.

From the geometry involved[1] it may easily be shown that h, the height in the atmosphere at which the solar illumination becomes inappreciable, is given by the equation

$$h = r(1 - \sec \tfrac{1}{2}T) ,$$

where T is the twilight arc and r the planet's radius. For Mars, h equals 13 km (8 mi.), far less than Wright's[2] value of 200 km (120 mi.).[3]

The density of the earth's atmosphere is approximately halved for every 5 km (3 mi.) ascended above the surface. For Mars, since the density varies inversely as the force of gravity, the corresponding figure will be 13 km (8 mi.). Thus the pressure at the Martian surface is only twice that at which the last trace of sunlight is visible.

Since the intensity of solar radiation on the earth is double that on Mars, it will be able to illuminate a rarer layer, therefore the density of the Martian atmosphere at the height h will probably be about twice the density of the corresponding layer of the terrestrial atmosphere. The pressure on the surface of Mars will be, therefore, four times the pressure in the earth's atmosphere at the height h_e, where, as seen from Mars, it ceases to be appreciably illuminated.

No direct method is available to evaluate h_e. If the pressure at the surface of Mars is taken to be 5 cm, the terrestrial pressure at h_e will be 1.25 cm, or, approximately, $h_e = 27$ km (16 mi.).[4]

It is well known that a sufficient quantity of air for deflecting an appreciable quantity of sunlight extends to a height of 63 km (39 mi.) above the earth's surface, where the pressure is only about one-hundredth that at h_e. As seen from Mars, the outermost layers would probably be invisible, so it is impossible to judge at what point h_e would lie, but the evidence that the 5-cm surface pressure is a maximum certainly seems to be substantiated. The minimum defined by the twilight arc is far less. From the foregoing it appears that the

[1] See Milham, *Meteorology*, diagram on p. 19.

[2] *Op. cit.*, p. 251.

[3] See my n. 1, added to proof in December, p. 58.

[4] *Smithsonian Physical Tables*, p. 421, Table 560.

pressure certainly is greater than one-hundredth, and probably greater than one-tenth, the indicated maximum. No weight should be attached to this evidence. Its real importance lies in showing that the height to which the atmosphere is visible by scattered sunlight is far less than Wright's value of 200 km.

The fact that persistent cloud effects, covering large areas, as observed by Lowell, Slipher, and others, are found on Mars is proof that the atmospheric density is great enough to support the fog, mist, or whatever it may be. In passing, it is sufficient merely to point out that the terrestrial pressure at a height of 15 km (9 mi.), above which level clouds are seldom found, is of the same order of magnitude as the maximum surface pressure for Mars.

Throughout this discussion, the implicit assumption has been made that the atmosphere of Mars is similar in constitution to that of the earth. There is no assurance that oxygen is present[1] but, for the purpose of scattering, nitrogen would suffice as well. If carbon dioxide is present in any large quantity, the mean molecular weight, and therefore the pressure of the atmosphere, would be inappreciably greater than here computed.

One possible experimental investigation which may reveal the minimum value suggests itself. The photographic albedo is measured at λ 4400 A. For the stated quantity of atmosphere, about half the total albedo, or 0.045, is due to the air. At λ 3600 A the scattering would be twice as great and the albedo should be about 0.13 instead of 0.09. A determination of this physical quantity at as short a wave-length as possible, by using a reflecting telescope, would probably provide an absolute criterion for determining a minimum limit to the atmosphere, for if the albedo of Mars does increase in the ultra-violet, it should certainly be attributed to scattering, while the absence of any increase would be conclusive evidence that the quantity of atmosphere is certainly much less than the maximum amount here quoted.

THE MARTIAN POLAR CAPS

From the fact that the polar cap appears very strong on the violet and very weak in the infra-red photographs, Wright suggests that it may possibly be counted an atmospheric phenomenon—perhaps a

[1] See my n. 2, p. 59.

fog or haze transparent to the infra-red but opaque to the ultra-violet. In the light of the preceding a different explanation may be advanced. The cap is probably merely the surface covering of snow and ice, as previously assumed. The intrinsic albedo of the surface in the red is so much greater than in the blue that the contrast between it and the more brilliant snow is much less marked. E. C. Slipher, who also interprets the phenomenon in this way, has pointed out that the observed changes in the appearance of the cap as it melts could not be accounted for if it were atmospheric.

FURTHER APPLICATION

Equation (2) will account for the color of the giant planets, particularly Uranus and Neptune. Their blueness is to be attributed, not to scattering, but to absorption. The spectra of both planets are crossed with heavy absorption bands in the red, thus explaining the color on the basis of an increase in x' for the red light. Since the absorption is beyond the range of either the effective vision or photographic wave-lengths, the blueness of these planets would probably not be revealed by their color-indices.

ACKNOWLEDGMENT

I wish to express my appreciation to Professor Henry Norris Russell for a very helpful conference during the preparation of this paper.

UNIVERSITY OF IOWA
February 1925

NOTE 1 (see p. 56).—Since the above was written, Wright has reduced his original estimates to approximately half (*Lick Observatory Bulletin*, No. 366). Even this new value (60 mi.) is more than could be allowed on the assumption of a perfectly scattering atmosphere. He suggests, however, that the photographs could be explained on the assumption that the Martian atmosphere is opaque to violet light. Scattering would not suffice in view of the small photographic albedo; the opacity, therefore, would have to be attributed to molecular absorption. This is, I believe, not in accord with certain physical principles, for a gas cannot, aside from scat-

tering, exert a general absorption over a wide region of the spectrum. The quantum theory demands that it take place in distinct bands. The spectrum of Mars apparently shows no indication whatever of these ultra-violet bands, which should exist for this planet in the same way that the red bands appear in the spectra of the giant planets. It is not likely that any of the gases which probably compose the Martian atmosphere would be more efficient in absorption than in scattering.

If an atmosphere as extensive as that found by Wright really exists, it would be necessary to assume the presence of a peculiar mist at a very high altitude (above almost the entire atmosphere), with the particular property of absorbing 90 per cent of the blue light and none of the red. A haze with properties such as this could hardly be constructed even artificially; neither clouds of dust nor of water-vapor would suffice, for the presence of either one would raise the albedo of the planet above the observed value.

NOTE 2 (see p. 57).—The recent note by Adams and St. John (*Publications of the Astronomical Society of the Pacific*, **37**, 158, 1925) is of interest in this connection. They have detected the presence of both water-vapor and oxygen. They state that "for equal areas, the water-vapor above the surface of Mars at the time of observation was of the order of 5 and the oxygen of the order of 15 per cent of that normally in the earth's atmosphere, that is, the oxygen was less than above Mt. Everest." Compare this result with the statement which occurs in the paragraph following equation (9) of the text. This shows the order of magnitude of the minimum quantity.

OHIO STATE UNIVERSITY
December 1925

PRELIMINARY ORBIT OF THE LONG-PERIOD SPECTROSCOPIC BINARY 37 ϕ^1 ORIONIS

By OTTO STRUVE

ABSTRACT

The orbital elements of the spectroscopic binary 37 ϕ^1 Orionis were derived from 56 spectrograms, covering an interval of 22 years. The period is 8.4 years; $\gamma = +33.2$ km/sec.; $K = 13.3$ km/sec.; $e = 0.22$; $\omega = 105°$; $a \sin i = 54$ million km. The calcium lines H and K show oscillations with the same period, but $\gamma = +16.2$ km/sec. and $K = 7.5$ km/sec. It was shown that these oscillations cannot be due to blending of a truly "stationary" line and a stellar line. The suggestion is made that it might be due to some sort of whirls in a calcium cloud, caused by the rapid motion of the components of a spectroscopic binary.

The radial velocity of 37 ϕ^1 Orionis was first determined by E. B. Frost and O. J. Lee from spectrograms obtained between 1903 and 1907. The average from these observations was about +40 km/sec., an unusually high velocity for a star of spectral type B. This in itself made it seem highly probable that the star was a spectroscopic binary, and in 1909 it was announced as having a variable radial velocity.[1] At the same time it was noted that the velocities derived from the calcium lines H and K differed systematically from those of the other lines, and the suggestion was made by E. B. Frost that the peculiar character of the lines in this star might be due to the existence of two separate doubles having two distinct velocity curves.

Observations of this star were carried on with the Bruce spectrograph, using the dispersion of one prism, at more or less irregular intervals. Since it was suspected from the beginning that the period was short and that the star was similar to ξ Persei or 9 Camelopardalis, which show periodic oscillations of the velocities of the calcium lines, the observers tried to obtain long series of plates during every season. The last series of this kind was obtained during the winter of 1922–1923.

The position of 37 ϕ^1 Orionis for 1900.0 is: $\alpha = 5^h29^m.3$; $\delta = +9°25'$. Its visual magnitude is 4.5 and its spectral type according to the *Henry Draper Catalogue* is Bo. Its spectrum contains many lines of good quality and the letter s should be added to the designa-

[1] *Astrophysical Journal*, **30**, 63, 1909.

tion of spectral type, as the lines are exceptionally sharp and narrow. It is one of the best spectra of class B and would be well suited to the accurate determination of stellar wave-lengths. The calcium lines H and K are both remarkably strong and sharp and should give very reliable results.

A total of 56 spectrograms was available for the investigation of the orbit. A preliminary discussion of the measurements of these plates showed that within a given season the velocities varied but little, although it seems probable that some of the differences in velocity obtained during the same season are real. However, all attempts to find the period of these oscillations have failed.

TABLE I

Date	No. of Plates	Vel. Star	Vel. *Ca*
		km/sec.	km/sec.
1903.8	2	+35.4	+17.3
1904.5	2	38.5	15.0
1907.5	2	43.5	18.4
1909.9	7	19.0	9.6
1910.4	4	22.9	8.2
1911.9	8	30.2	21.3
1912.9	8	31.6	12.9
1913.1	4	29.4	21.4
1917.3	1	26.7	.....
1922.8	17	46.2	23.0
1925.5	1	35.9	4.8

An inspection of the measures shows at once that there are also slower changes of velocity present. The original measures will soon be published in connection with a complete study of the radial velocities of 368 stars of spectral type B. Table I contains only the average velocities for each season. The probable error of one observation is about ± 3.5 km/sec., while the probable error for each seasonal velocity, except the last one, averages about ± 1.0 km/sec. The last velocity in Table I is based on one plate only and has a correspondingly smaller weight.

In view of the fact that the calcium lines H and K always give systematically smaller velocities than the other lines, the measurements of these lines were not included in the means of the star's velocity and are given separately in the last column of Table I.

The velocities collected in Table I can all be represented by a

period of 8.4 years. If the dates are reduced to the same epoch, separate velocity curves are obtained for the star and for the calcium vapors. Each curve was independently used for the determination of the orbital elements. In order to obtain a better representation, two different graphical methods were used for each curve. The standard method of Lehmann-Filhés was used at first. Since the eccentricity was found to be appreciable ($e=0.2$), it seemed advisable to use Schwarzschild's method for the determination of the

TABLE II

Element	Lehmann-Filhés	Laves	Adopted
	Star		
P	8.4 years	8.4 years	8.4 years
γ	+33.2 km/sec.	+33.2 km/sec.	+33.2 km/sec.
e	0.19	0.25	0.22
K	13.3 km/sec.	13.3 km/sec.	13.3 km/sec.
ω	105°.8	103°	105°
T	1908.3	1908.2	1908.3
$a \sin i$	54 mill. km	.....	54 mill. km
	Calcium Lines		
P	8.4 years	8.4 years	8.4 years
γ	+16.2 km/sec.	+16.2 km/sec.	+16.2 km/sec.
e	0.22	0.28	0.25
K	7.5 km/sec.	7.5 km/sec.	7.5 km/sec.
ω	107°.0	108°	107°
T	1908.3	1908 3	1908.3
$a \sin i$	30 mill. km	.....	30 mill. km

time of periastron passage. The rest of the elements were then derived according to the method of K. Laves in which the hodograph circle is used.[1]

The two determinations give closely the same results. Table II contains the elements derived by each method separately. The last column contains the values which were finally adopted. The curves in the accompanying figure were computed by means of these elements.

The close resemblance between the shape of the velocity curves for the star and for the so-called stationary calcium lines is very

[1] *Astrophysical Journal*, **26**, 164, 1907.

conspicuous (Fig. 1). The period is the same, but the amplitude is only about one-half that of the other lines. The velocity of the center of mass is quite different: +33.2 km/sec. for the star and +16.2 km/sec. for the calcium lines.

J. S. Plaskett[1] and others have suggested that the oscillations of the "stationary" calcium lines could be explained as an effect of blending of a truly stationary line with one belonging to the star

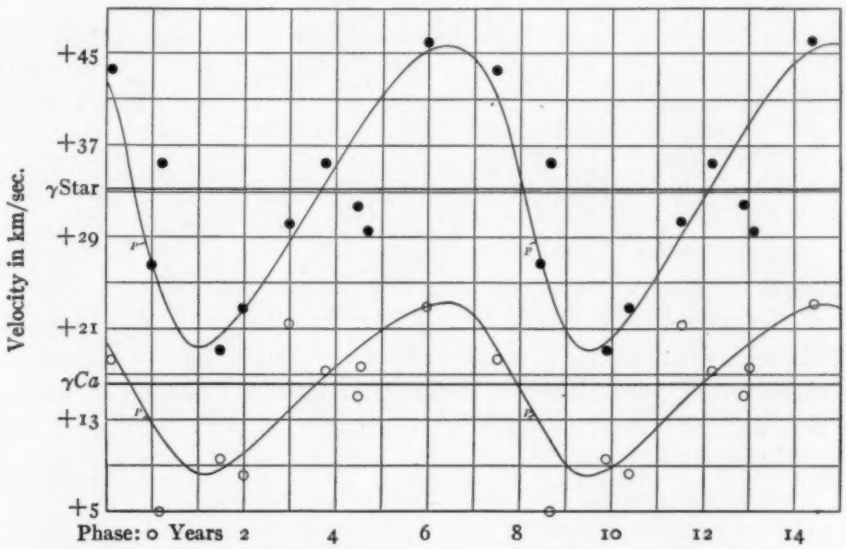


FIG. 1.—Velocity curve of the spectroscopic binary 37 ϕ^1 Orionis. ●, velocity curve of the double star; ○, velocity curve from calcium lines H and K.

itself. On the other hand, R. K. Young[2] has suggested that perhaps the calcium lines represent the real velocity of the star, while the other lines are displaced as a result of causes other than the Doppler effect. In fact, Young thought that the large velocity which ϕ^1 Orionis had at that time (+40 km/sec.) was due to such a spurious shift. Such an explanation would seem possible in the case of stars having unusually broad lines, as ξ Persei or δ Orionis. The lines of ϕ^1 Orionis, however, are sharp and well defined, and there is no reason to suspect any such effect as an unsymmetrical broadening

[1] *Publications of the Dominion Astrophysical Observatory*, Victoria, **2**, 344, 1924.

[2] *Publication of the Dominion Observatory*, Ottawa, **4**, 26, 1916.

of the lines, which could be attributed to upward or downward currents in the atmosphere of the star. Besides, the γ-velocity of ϕ^1 Orionis, +33.2 km/sec., is not excessive as compared with the velocities of other stars in the same region.

It is important to note that the velocities of star and calcium in 37 ϕ^1 Orionis are almost identical with those found here for the double star 39 λ Orionis, which is situated $0^m.3$ east and $27'$ north from ϕ^1. These velocities are given below:

	α	δ	Spec.	Mag.	Vel. Star	Vel. *Ca*
37 ϕ^1 Orionis	5 29.3	9°25′	Bo	4.5	+33.2	+16.2 km/sec.
39 λ Orionis br....	5 29.6	9 52	Oe5	3.7	+31.8	+14.1
39 λ Orionis ft.....	5 29.6	9 52	B1	5.6	+30.0	.....

The close agreement of these velocities, further supported by a good agreement in the proper motions, leads us at once to suspect equal and parallel motions for these two stars in space. The equality of the calcium velocities indicates that it is the same cloud which envelopes both stars. If we subtract from these velocities the component due to the motion of the solar system (+15.3 km/sec.), we find that the two stars are moving with a velocity of +17 km/sec., while the calcium cloud is practically at rest with respect to the stellar system. This would mean that the effect of the stellar calcium line in a supposed blend is completely negligible and that no blending takes place. If we use only the values derived for 37 ϕ^1 Orionis, we find the following velocities with respect to the stellar system:

	km/sec.
37 ϕ^1 Orionis, star.....	+17.9
Calcium	+ 0.9
Solar motion	0.0

This, then, would mean a blending of the "stationary" and the "stellar" lines in the ratio 0.9 to 17.5, or 0.05.

We find an entirely contradictory result, if we determine the effect of blending from the amplitudes of the velocity curves.

If we suppose that I^* and I_{Ca} are respectively the intensities of the lines due to the star and to the calcium cloud, then it is evident that

$$\frac{I^*}{I_{Ca}} = \frac{\gamma_{Ca}}{\gamma^* - \gamma_{Ca}} = \frac{K_{Ca}}{K^* - K_{Ca}}.$$

The first equation gives, as we have seen:

$$\frac{I^*}{I_{Ca}}=\frac{0.9}{17.5-0.9}=0.05\,.$$

If we use the second equation, and put $K^*=13.3$ km/sec.; $K_{Ca}=7.5$ km/sec., we obtain:

$$\frac{I^*}{I_{Ca}}=\frac{7.5}{13.3-7.5}=1.3\,.$$

This value is twenty-six times larger than the one obtained from the γ velocities.

This seems to be a proof that the oscillations of the "stationary" calcium lines which occur usually with the same period, but with a smaller amplitude and different velocity of the system, are not due to the effect of blending.

On the other hand, the difference in γ is so striking that one is led to accept definitely the existence of vast calcium clouds in space. The mechanism which produces the ionization of calcium in the vicinity of the hottest stars[1] is still a mystery.

It is not impossible that the rapid motion of a binary star passing through such a cloud produces something like a whirl in the material of which the cloud is made, similar to whirls of water caused by the propeller of a boat in rapid motion. Such whirls may produce the effects observed in such stars as ϕ^1 Orionis and others. They would evidently depend on the velocity of the components of the binary, in other words, on the value of K. This seems to be true, since in every case

$$K_{Ca}<K^*\,.$$

On the other hand, the relation would depend also on the distance at which the star is capable of ionizing the cloud, and finally on the density of the cloud itself.

The residuals O. – C., as can be seen from the figure, are large and seem to show that the supposition, that smaller and shorter oscillations exist, is justified. Besides, there is considerable uncertainty

[1] J. S. Plaskett, *Publications of the Dominion Astrophysical Observatory*, Victoria, 2, 342 ff., 1924.

involved in the determination of the period. Therefore I have not attempted to improve the orbit by the method of least squares.

The value of $a \sin i$ is 54 million km. Kapteyn[1] found for the helium stars in the constellation Orion, but situated outside the "Nebula-group," a parallax of $0''.008$. Assuming this to be correct and supposing that i is large, such that $a = 60$ million km and that $M_1 = M_2$, we would obtain for the semi-major axis of the visual orbit $a = 0''.006$. This value is so small that it appears rather hopeless that the star will be discovered visually even with the use of a large interferometer.

YERKES OBSERVATORY
August 12, 1925

[1] *Astrophysical Journal*, **47**, 1918; *Contributions from the Mount Wilson Observatory*. No. 147, p. 20, 1918.

INTERNAL MOTIONS OF MESSIER 33[1]

By KNUT LUNDMARK

ABSTRACT

Internal motions of Messier 33.—The photographs measured and discussed by van Maanen, *Mt. Wilson Contr.* No. 260, were remeasured with the same measuring instrument and discussed by the same method. The same comparison stars and nebular points were selected for measurement. Any systematic difference in results is therefore of personal origin.

The values found for the *proper motion* are $\mu_\alpha = -0''.0015$, $\mu_\delta = -0''.0050$; van Maanen's results are $+0''.0034$ and $-0''.0044$, respectively. The annual *rotational components* can be represented by $-0''.00084 + 0''.00032\,\rho$, and $+0''.01063 + 0''.00119\,\rho$ (van Maanen), ρ being the radial distance in minutes of arc. For the mean distance of the 399 nebular points, $\rho = 7'.6$, these expressions for the rotational component become $+0''.00160$ (Lundmark) and $+0''.01969$ (van Maanen). The *correlation coefficients* between the rotational and radial components found by the two measures are $r_{rot.} = 0.282 \pm 0.046$ (m.e.), $r_{rad.} = 0.453 \pm 0.040$ (m.e.).

During a stay at the Mount Wilson Observatory in 1922–1923, at the suggestion of Director W. S. Adams and Dr. Adriaan van Maanen, I measured the well-known spiral nebula Messier 33 (N.G.C. 598) for internal motion. The plates used were the same as those measured by van Maanen,[2] and, for purpose of comparison, the same objects were selected for measurement. Thus four hundred nebular points, which as a rule are starlike objects, well distributed in the spiral arms and having a considerable range in brightness, were measured by both of us. The same instrument was used, the new large stereocomparator with a monocular arrangement, and the precautions suggested and described by van Maanen were observed throughout.[3] My measures were made at the same time as those of van Maanen, but the reduction was carried out after my return to Upsala.[4]

The plates were measured in six positions, three in each equatorial co-ordinate. For each set, the twelve plate constants were derived by a least-squares solution of the equations of condition

[1] *Contributions from the Mount Wilson Observatory*, No. 308.

[2] *Mt. Wilson Contr.*, No. 260; *Astrophysical Journal*, **57**, 264, 1923.

[3] *Mt. Wilson Contr.*, No. 118; *Astrophysical Journal*, **44**, 210, 1916; *Mt. Wilson Comm.*, No. 72, 1921; *Proceedings of the National Academy of Sciences*, **7**, 1, 1921.

[4] Van Maanen, *Mt. Wilson Contr.*, No. 260; *Astrophysical Journal*, **57**, 264, 1923; *Mt. Wilson Annual Report*, 1923.

yielded by the twenty-four comparison stars. The values of μ_α and μ_δ of the nebular objects were obtained with the aid of these plate constants, and the results from the different sets of measures were combined to form mean values. In discussing the measures, one point, No. 367, was omitted because its motion ($\mu = 0''.112$ annually) is so large that it cannot well be a part of the nebula. By different methods, the motion of translation—the total proper motion of the nebula—was found to be

$$\mu_\alpha = -0''.0015 \; ; \quad \mu_\delta = -0''.0050 \, ,$$

which is in good agreement with the corresponding values of van Maanen, namely, $+0''.0034$ and $-0''.0044$, respectively.

Subtracting the translational motion from the values of μ_α and μ_δ for the nebular objects, we obtain the internal motions, which are illustrated in Plate III. These have been resolved into rotational and radial components. The positive sign is used for motions in the directions NESW and outward in the nebula.

The mean values of these components are

$$\mu_{\text{rot.}} = +0''.0016 \; ; \quad \mu_{\text{rad.}} = -0''.0057 \, .$$

The corresponding values from van Maanen's measures are $+0''.0196$ and $+0''.0029$, respectively. The standard deviations or dispersions around the means are

$$\sigma_{\text{rot.}} = \pm 0''.0065 \, , \quad \sigma_{\text{rad.}} = \pm 0''.0091 \text{ (Lundmark)} \, ;$$
$$\sigma_{\text{rot.}} = \pm 0''.0080 \, , \quad \sigma_{\text{rad.}} = \pm 0''.0096 \text{ (van Maanen)} \, .$$

The rotational motions found by me can be represented by the linear expression

$$\mu_{\text{rot.}} = -0''.00084 + 0''.00032\rho \, ,$$

where ρ is the distance from the center of the nebula in minutes of arc.

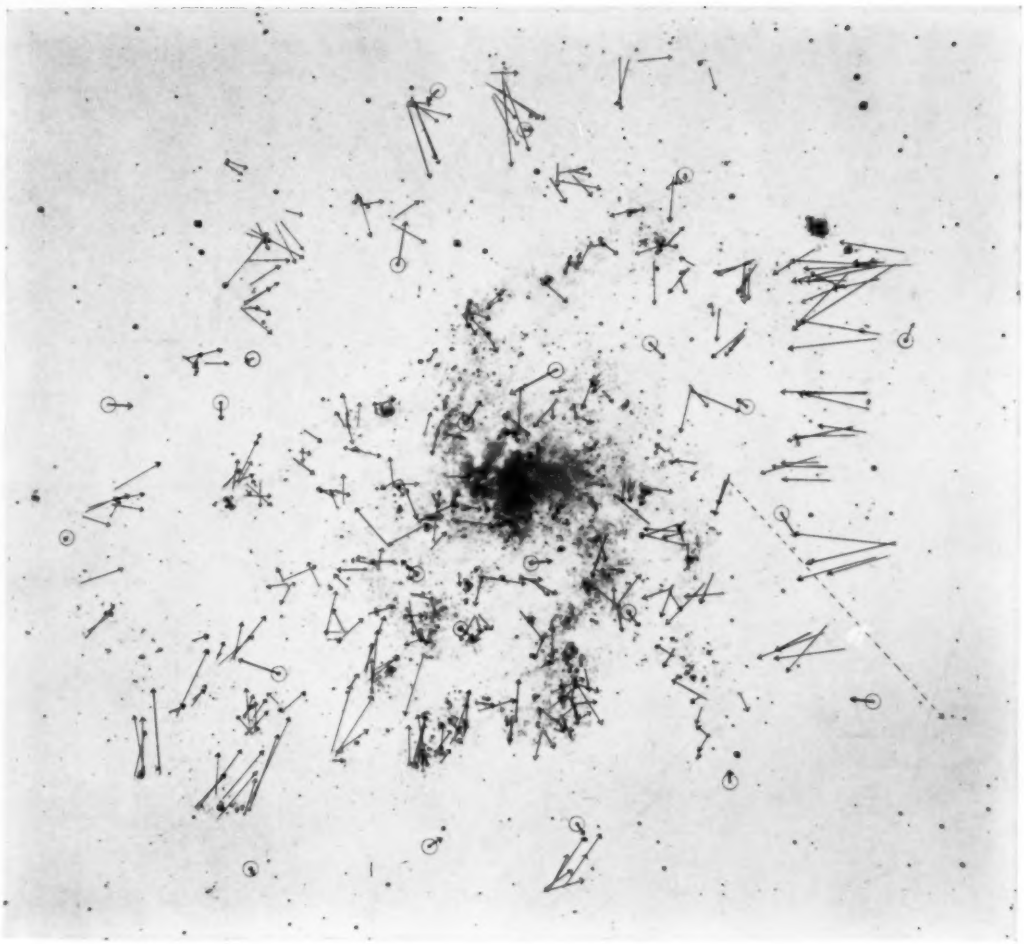
A similar solution of van Maanen's components gave

$$\mu_{\text{rot.}} = +0''.01063 + 0''.00119\rho \, .$$

PLATE III

N

W



E

INTERNAL MOTIONS IN MESSIER 33

The arrows indicate the directions and magnitudes of the annual motions. The comparison stars are inclosed in circles. The scale of the nebula is 1 mm = 12″.4.

UNIV. OF MICH.

Since most of the nebular points are at some distance from the center, both series of measures give a general rotational motion of the nebula in the same direction. Further, both series show an increase in the rotation from the center outward of the same general order of magnitude. Using as variates the values of $\mu_{\text{rot.}}$ found by each measurer and computing the correlation coefficient r, we have

$$r = 0.282 \pm 0.046 \text{ (m.e.)},$$

a value which, according to the rules used in statistics, should indicate a real correlation.

As the mean distance ρ of the 399 objects is $7'.61$, the mean period of rotation will be 2.8×10^6 years according to my measures, and 0.23×10^6 according to the measures of van Maanen.

The correlation between the radial motions of the two measures is very definite. We have

$$r = +0.453 \pm 0.040 \text{ (m.e.)}.$$

When the radial motions were plotted against ρ, a rather definite relation was indicated. A least-squares solution was accordingly made, whereby it was found that the values of $\mu_{\text{rad.}}$ could be represented by the formula

$$\mu_{\text{rad.}} = -0''.00117 + 0''.00094\rho - 0''.00017\rho^2.$$

The solution reduced the sum of the squares of the residuals from 45,845 (unit $= 0''.001$) to 25,623, or 44 per cent.

It does not seem very plausible that the change in $\mu_{\text{rad.}}$ with distance from the center should express any phenomenon arising from the actual internal motions of the nebula. It is perhaps more reasonable to assume its cause to be instrumental or optical, and the following explanation is tentatively suggested. The older plate, taken by Ritchey, is of unsurpassed quality and shows excellent definition over relatively a wide field. The later plate is very good, indeed, but does not quite match the older plate. Now it must be remembered that the comparison stars, as a rule, are brighter than the nebular objects, because the stars had to be so selected that there

was little probability that they formed part of the nebula. Thus it is possible that a differential effect of coma has crept in and affected my measures in such a way as to introduce a spurious inward motion.

Since such an effect of coma would be radial, it leaves the foregoing conclusions regarding the rotation of the nebula unchanged; but it certainly must be of importance when analyzing the motion along and across the spiral arms.

The radial motions have been corrected for the general increase from center outward according to the foregoing formula. The corrected values of $\mu_{rad.}$ have been combined with the unchanged $\mu_{rot.}$. The resultant motion is thought to represent the combined effect of internal motion and unavoidable errors of measurement. This has been resolved into components along the spiral arms, μ_{stream}, and perpendicular to the arms, $\mu_{trans.}$. The positive sign is used for motions from the center outward and for motions toward the left for an observer facing outward along the spiral arms.

The mean stream and transverse components are

$$\mu_{stream} = +0''.0018 , \quad \mu_{trans.} = -0''.0004 .$$

Van Maanen found the values $+0''.0205$ and $-0''.003$, respectively. The correlation between the two series of stream motions is expressed by

$$r = +0.330 \pm 0.047 \text{ (m.e.)} .$$

The stream motions derived by me show an increase with increasing distance from the center of more or less the same order of magnitude as that found by van Maanen.

Comparing van Maanen's results for the internal motions of seven spiral nebulae with the rotation of the spirals N.G.C. 224, 3031, and 4594, spectroscopically measured, we find, according to different assumptions, mean distances ranging between 10,000 and 40,000 light-years. The measures of Messier 33 made by me suggest somewhat larger distances, varying between 40,000 and 160,000 light-years. On account of the few cases investigated by spectroscopic methods, considerable uncertainty is involved in the application of the method.

The following formula[1] is easily deduced:

$$M = [3.71856]\mu^2_{\text{rot}} d^3 \tan \rho ,$$

where M is the mass within the distance ρ from the center, expressed in minutes of arc, and d, the distance of the nebula from us, in parsecs. If we use the distance of 300,000 parsecs, as suggested by Hubble[2] from his investigations of Cepheids, the following values of M are obtained:

$$1.6 \times 10^{14} \odot \text{ (van Maanen) ;}$$
$$1.1 \times 10^{12} \odot \text{ (Lundmark) .}$$

I wish to express my sincere thanks to Dr. Adams and Dr. van Maanen for the opportunity given me to measure the plates of Messier 33. My thanks are also given to them, as well as to other members of the staff, for the interest taken in my work and for valuable help and suggestions during the memorable time I had the privilege of working at the Mount Wilson Observatory.

UPSALA
August 1925

[1] *Publications of the Astronomical Society of the Pacific*, **34**, 108, 1922.

[2] *Observatory*, **48**, 139, 1925.

ERRATA

Vol. **62,** No. 4, November, 1925, article on: "The Law of Sun-Spot Polarity," by George E. Hale and Seth B. Nicholson:

Page 270, in the Abstract, eighth line, *for* Tables III, IV, and V *read* Tables I, II, IV, and V.

Page 271, eighth line from bottom, *for* field-stength *read* field-strength.

Page 291, fifth line from top, *for* as follows *read* given in Table III.

Page 294, twelfth line from bottom, *for* spots *read* spot.

Page 299, fifth line from top, *for* altitudes *read* latitudes.